Mathematical Modelling of a Deregulated Electricity Market With Different Production Capacities

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1 Problem Statement

We consider as basis for our work, the model of a deregulated electricity market presented in von der Fehr and Harbord (1992a), as follows:
Let g_1, g_1, ..., g_N be the electricity generator firms in the market; assume that each g_i can produce at most k_i electricity units per day, thus, let $K = \sum_{i=1}^{N} k_i$ be the daily production capacity of the whole system. Denote by c_i the unit production cost of the firm g_i, and assume that the daily electricity demand d is a random variable which takes values in the set $\{1, 2, \ldots, K\}$ (leading to disregard shortage cases) according to some probability distribution $\pi_j = Pr(d = j)$ for $j \in \{1, 2, , \ldots, K\}$.
Now, the market operating manner is the following. Each day t, each generator firm g_i sends to a coordinator entity, in a sealed bid way, the price $p_i \in [0, \bar{p}]$ in which the firm is willing to sell each electricity unit produced for the day $t + 1$, being $\bar{p}$ the highest price that a firm can offer. After that, at the end of the day t, the coordinator makes a low-high ranking with the prices offered by the firms; it is considered as a bijection $r : \{1, 2, \ldots, N\} \to \{1, 2, \ldots, N\}$, which satisfies that $r(n) < r(m) \Rightarrow p_n < p_m$. Given that there could be many rankings which satisfies the property above, the coordinator randomly selects one of the possible rankings with the same probability $\frac{1}{R(p_1,p_2,\ldots,p_N)}$, where $R(p_1, p_2, \ldots, p_N)$ is the number of the possible rankings for $p_1, p_2, \ldots, p_N$. Once the ranking was elected, denote by n_i the index of the firm which has the position i in the ranking, thus, the coordinator observes the demand d for the day and lets the first $\rho - 1$

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firms supplying their whole capacity, and the firm ρ, supplying $d - K_{\rho-1}$, where $K_j = \sum_{m=1}^{j} k_{n_m}$ and $\rho = max\{j : K_{j-1} < d\}$. Finally, each firm is paid p_{n_ρ} and the auction for the next day begins again, hence, the utility in a day for the firm g_i is given by:

$$u_i = \delta_\rho(i)(d - K_{\rho-1})(p_{n_\rho} - c_{n_\rho}) + \sum_{m=1}^{\rho-1} \delta_m(i) k_{n_m}(p_{n_\rho} - c_{n_m})$$

where $\delta_m(i) = 1$ if $i = n_m$ and $\delta_m(i) = 0$, otherwise.

Then, we have that utilities depend on prices offered, demand, and the ranking (if there are ties), so, we can see this game as a random phenomenon, with sample space given by:

$$\Omega = \{(p_1, p_2, ..., p_N, d, r) : p_n \in [0, \overline{p}], d \in \{1, 2, ..., N\}, \text{ r ranking for } (p_1, p_2, ..., p_N)\}$$

and a probability density function considered with the form:

$$f(p_1, p_2, ..., p_N, d, r) = f_1(p_1) \cdot f_2(p_2) \cdot ... \cdot f_N(p_N) \cdot \pi_d \cdot \frac{1}{R(p_1, p_2, ..., p_N)}$$

where $f_1, f_2, ..., f_n$ are the density probability function for the unitary price offered by each firm (the mixed strategy played by each player).
Now, given that it is supposed that firms act rationally, we have to work in a Nash equilibrium situation, thus, adapting the definition of Nash equilibrium in finite mixed strategies given by Navarro et al. (2003), we have that $f_1, f_2, ..., f_n$ is a Nash equilibrium if

$$E_{f_1, f_2, ..., f_n, ..., f_N}(u_n) \geq E_{f_1, f_2, ..., \tilde{f_n}, ..., f_N}(u_n)$$

where $\tilde{f_n}$ is any other possible density probability function for the price offered by g_n.
Now, it is known (adapting Theorem 3.1 in Navarro et al. (2003)) that $f_1, f_2, ..., f_n$ is a Nash equilibrium if and only if for any n, the expected profit $\Phi_n(p)$ of firm g_n given that it plays the pure strategy $p_n = p$ and the other $g_i's$ play $f_i's$, is independent of p. Thus, a first problem appears, it is to find an explicit formula for $\Phi_n(p)$, which is necessary if we want to obtain the equilibrium strategies $f_1, f_2, ..., f_n$ (or better, their cumulative distributions associated, $F_1, F_2, ..., F_n$), even if they are obtained numerically; that is the main focus of this research.

2 Objectives

2.1 General Objective

Find an explicit formula for $\Phi_n(p)$ in the case $N = 4$.

2.2 Specific Objectives

- Understand each component of the explicit formula of $\Phi_n(p)$ in the case where all firms have the same properties.
- Define the pure strategy space and mixed strategy space for each one of the players.
- Propose a structure of the formula for $\Phi_n(p)$, when $N = 4$ using ideas of the previous case.

3 Preceding Research

A first approach to deregulated electricity markets from game theory is given by Richard J. Green (1992), who consider the British case as a duopoly, giving some empirical simulations of electricity market, making some observations about a possible equilibrium of the market. After that, von der Fehr and Harbord (1992b,a) give a more theoretical treatment of previous duopoly with some generalizations in multiple firms case.
More recently, in Colombia, the problem of deregulated electricity markets has been treated from several points of view, like agent based models (Gallego et al., 2008) and artificial intelligence (Quintero et al., 2014). Actually, some researchers of EAFIT University are working on this problem and this project is the continuation of the work done in Cadavid et al. (2016), where we considered a very simple case of this market type.

4 Justification

The 1990s featured a wave of deregulation and privatization of electricity industries in several nations (Grilli, 2010), therefore the study of markets involved in this industry has become important for these countries. Now, given the relevance which has the electric sector in most of the countries, it is important to study the characteristics of the electricity market, such as the expected aggregated supply curve in equilibrium cases.
If we want to be able to construct these curves in the cases where generator firms have different production capacities and there is an equilibrium, is necessary to make simulations of the auctions, but this is not possible if we do not have the equilibrium strategies played by each firm, and those are obtainable if we know an explicit form of $\Phi_n(p)$. Now, finding that formula becomes a very complex probability problem if we do not know anything about how could it be, then, looking for that formula in some representative case ($N = 4$) which tells us how the general formula looks is an useful step for this research.

5 Scope

The actual aim of the general project of professor Cadavid and his colleagues is to study the expected aggregated supply curves in deregulated electricity markets where the generator firms have different production capacities. This work is an intermediate step for the research, which consists in giving a formula for $\Phi_n(p)$ in the case $N = 4$, with the aim of make easier the search of the general formula, which is essential for the general aim.

6 Methodology

The realization of this project will take five weekly hours, it includes one weekly meeting with the tutor for making questions, proposing ideas and sharing progress; also, it includes the acquisition of some probability and game theory basis, and the time dedicated for reports and presentations.
The steps to follow in order to reach the general objective are as follows:

- Acquire some specific basis of probability theory and game theory.
- Define the strategy space for each player (firms, coordinator and nature).
- Study the formula for $\Phi_n(p)$ in the case where all generator firms are identical.
- Propose a formula for $\Phi_n(p)$ when the market is composed by $N = 4$ firms possibly different.

7 Activity Schedule

A specific activity schedule is presented in Table 1.

Week	**Activity**
3, 4	Elaboration of proposal report and presentation.
5,6	Acquirement of some missing basis of probability theory and game theory.
7,8	Define the strategy space of each player.
9	Study the formula for $\Phi_n(p)$ in the simplest case.
10	Elaboration of progress presentation.
11-14	Find the formula for $\Phi_n(p)$ in the case $N = 4$.
15,16	Elaboration of project report.
17,18	Improvements of the formula or extra work.
19	Elaboration of project presentation

Table 1: Activity Schedule

8 Budget

This project will not require any financing.

9 Intellectual Property

Results obtained in this project are property of the group of researchers which are carrying out the general project which this one come from, and EAFIT University, according to the intellectual property regulation (EAFIT, 2009).

References

N.-H. M. von der Fehr and D. Harbord, "Spot market competition in the UK electricity industry," Department of Economics, University of Oslo, Memorandum no. 9, 1992.

J. Navarro, E. Tena, and J. Pastor, *Teoría de Juegos.* Editorial Alhambra S. A. (SP), 2003.

D. M. N. Richard J. Green, "Competition in the British electricity spot market," *Journal of Political Economy*, vol. 100, no. 5, pp. 929–953, 1992.

N.-H. M. von der Fehr and D. Harbord, "Spot market competition and long term contracts: The case of a deregulated electricity industry," *University of Oslo*, 1992.

L. Gallego, O. Duarte, and A. Delgadillo, "Strategic bidding in Colombian electricity market using a multi-agent learning approach," in *Transmission and Distribution Conference and Exposition: Latin America, 2008 IEEE/PES*, Aug 2008, pp. 1–7.

O. L. Quintero, L. Hurtado, and J. J. García, "Estimación del precio de oferta de la energía eléctrica en Colombia mediante inteligencia artificial," *Revista de Métodos Cuantitativos para la Economía y la Empresa*, vol. 18, pp. 54–87, 2014.

C. Cadavid, M. E. Puerta, J. D. Vélez, and J. F. García, "q-exponential behavior of expected aggregated supply curves in deregulated electricity markets," *ArXiv e-prints*, Jan. 2016.

L. Grilli, "Deregulated Electricity Market and Auctions: the Italian case," Dipartimento di Scienze Economiche, Matematiche e Statistiche, Universita' di Foggia, Quaderni DSEMS, 2010.

U. EAFIT, *Reglamento de Propiedad Intelectual*, 2009.