Research Practice 2
Proposal report

Heuristic and exact solution strategies for the Team Orienteering Problem

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1 Problem Statement

The team orienteering problem (TOP) is a generalization of the orienteering problem (OP), where m vehicles are available to visit n nodes and the goal is to determine m routes, without exceeding given thresholds, that maximize the total collected prize, due to each node has a specific profit. No node can be visited more than once by one or several routes and there is the possibility of not visiting all nodes.

The TOP can be modeled as a multi-level optimization problem. At the first level, it is necessary to select a subset of points for the team to visit. At the second level, it is necessary to assign points to each member of the team in order to maximize the total profit. At the third level, it is necessary to construct a feasible route through those points to each member [1].

This problem can be model as a linear integer programming as [2] shows. In this case, the decision variables are y_{ik}, which is a binary variable that represent whether the node is visited or not, and x_{ijk}, which represents the number of times that the arc (i, j) is crossed by the k vehicle.

Its parameters are:

- d_{ij}: distance between the nodes.
- s_i: service time on the node i.
- V: set of nodes.
- U: subset of V.

$$max \sum_{i=1}^{n-1} \sum_{k=1}^{m} r_i y_{ik} \quad (1)$$

$$s.t. \sum_{j=1}^{n-1} \sum_{k=1}^{m} x_{0jk} = 2m, \quad (2)$$

$$\sum_{i<j} x_{ijk} + \sum_{i>j} x_{jik} = 2y_{jk} (j = 1, 2, ..., n-1; k = 1, 2, ..., m), \quad (3)$$

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$$\sum_{i=0}^{n-1}\sum_{j>i} d_{ij}x_{ijk} + \sum_{i=1}^{n-1} s_i y_{ik} \leq L(k=1,2,...,m), \tag{4}$$

$$\sum_{k=1}^{m} y_{ik} \leq 1(i=1,2,...,n-1), \tag{5}$$

$$\sum_{i,j\in U} x_{ijk} \leq\mid U\mid -1(U\subset V\setminus\{0\}; n-2\geq\mid U\mid\geq 2; k=1,2,...,m), \tag{6}$$

$$x_{0jk}\in\{0,1\}(j=1,2,...,n-1;k=1,2,...,m), \tag{7}$$

$$x_{ijk}\in\{0,1\}(1\leq i\leq j\leq n-1;k=1,2,...,m), \tag{8}$$

$$y_{ik}\in\{0,1\}(i=1,2,...,n-1;k=1,2,...,m), \tag{9}$$

Objective function is given by (1) which is to maximize the total profit. (2) ensures that there is m routes strating at node 1 and finishing at node n. The connectivity of each node is checked on (3) and the time limit of each route is verified on (4). (5) ensures that each node is visited at the most ones. The sub-tours are forbidden by (6). Finally, (7) to (9) are requires for the decision variables to be binary.

Given the specifications above, it will be developed two different methodologies to approach an optimal solution: a) based on exact mathematical programming methods, such as mixed integer linear programming (MILP) and constraint programming (CP), and b) based on hybrid heuristic methods using mathematical models.

2 Objectives

2.1 General Objective

To compare exact solution approaches and propose a heuristic algorithm based on the hybridization of mathematical programming formulations and adaptive large neighborhood search heuristics (ALNS) to find a solution of TOP.

2.2 Specific Objectives

- To describe and propose several mathematical formulations for the TOP.
- To solve the described mathematical models by using a robust software (for instance IBM Ilog Cplex [3]).
- To propose and develop a hybrid heuristic algorithm based on ALNS using mathematical programming formulations during the construction phase.
- To compare the performance of both exact and heuristic solution approaches on small instances.

3 Justification

Constraint programming is used on computer science and artificial intelligence to find solutions to scheduling, routing and combinatorial optimization problems.

“The essence of constraint programming is a two-level architecture integrating a constraint and a programming component. The constraint component provides the basic operations of the architecture and consists of a system reasoning about fundamental properties of constraint systems such as satisfiability and entailment. The constraint component is often called the constraint store, by analogy to the memory store of traditional programming languages. Operating around the constraint store is a programming-language component that specifies how to combine the basic operations, often in non-deterministic way” [4].

CP has some advantages over MILP such as allowing any expression over the variables, for example:

$$x^3(y^2 - z) \geq 25 + x^2 \cdot \max(x, y, z)$$

thus, CP models are more intuitive, close to natural language. Also the domain of a decision variable can be any possible set, for example it could be a set of integers or people names. There is no restriction on the type of each decision variable, so decision variables can take on integer values, real values, set elements, or even subsets of sets.

This programming type has had great results in some other vehicle routing problems such as travelling salesman problem (TSP) and capacitated vehicle routing problem (CVRP), but it has not been implemented in TOP. Therefore, the aim of this project is to take advantage of the benefits given by CP in order to find optimal solutions on small instances to TOP efficiently.

On the other hand, heuristic methods are vast used in the solutions of the vehicle routing problems with great success. Particularly, it will be implemented a variations of ALNS, which is based on the concepts of destroy and repair. The ALNS was first proposed by Pisinger and Ropke [5], and it is composed by a set of procedures to destroy and repair in order to improve a single solution. In this project the repair procedures will be replaced by a mathematical model, while several destroy procedures will be used to reach different final solutions. The proposed repair procedure strategy based on mathematical formulations is where the heuristic that will be designed is going to differ form the heuristic algorithms in the literature. This kind of heuristic strategy belongs to a recent family of hybrid methods known as “matheuristic”.

4 Scope

The aim of the project is to compare different mathematical programming paradigms, CP, MILP and heuristic algorithms, when solving the team orienteering problem on several benchmark instances.

5 Preceding Research

Golden et al. [6] show that the orienteering problem, which is a special case of the TOP, is NP-Hard. That means that this kind of problems, OP and TOP, would need to use heuristic algorithms to solve large instances.

The first formal approach for the TOP is developed in Chao et al. [1]. Also, it presents the first metaheuristic proposed to solve the TOP with different data sets. In Archetti et al. [7] two different heuristic methods are presented, variable neighbor search (VNS) and tabu search (TS), to solve the TOP. It was found that the VNS method is more effective and efficient than the other one. In fact, the VNS method is better than the heuristic presented in [1]. Furthermore, in Ke et al. [8] it is shown another heuristic solution strategy based on ant colony optimization (ACO), to solve our vehicle routing problem, TOP. Although the ACO algorithm was promising, it did not improve the former results.

Regarding CP, we have not found any publications of this paradigm to solve the TOP yet. However, it has been used on other vehicle routing problem. For instance, Shaw [9] presents a Large Neighborhood Search (LNS) combined with CP obtaining better results than the ones on the literature for the capacitated vehicle routing problem (CVRP) by Christofides et al. [10] instances, and the vehicle routing problem with time windows (VRPTW) by Solomon [11] instances. Also, in Backer et al. [12] is presented a combination of CP and a Tabu Search heuristic algorithm, as the former algorithm, which also found several new best solutions on the Solomon [11] instances.

6 Methodology

The development process of the project will have different stages. The first stage deals the understanding of the team orienteering problem, its specific restrictions and features, so it will be easier to understand the difference between the mathematical models which represent it. Then, mathematical models based on MILP and CP will be formulated in order to solve the target problem. The mathematical models will be implemented on Gurobi or IBM Ilog Cplex solvers in order to find exact solutions on small benchmark instances.

Afterwards, it will be designed a matheuristic algorithm based on mathematical programming to find solutions to this problem. Finally, the performance of both methodologies will be compared and analyzed to conclude the advantages and disadvantages of each approach.

7 Schedule

On Table 1 are specified the time in weeks that we are going to take for each part of this research in order to achieve the objectives described above.

Table 1 Schedule

Activity	*Weeks*																	
	1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*
State of the art	■	■	■	░	░	░	░	░	░	░	░	░	░	░	░	░	░	░
MILP Model				■	■													
CP Model						■	■	■										
Math models Comparison									■	■								
Matheuristic											■	■	■					
Matheuristic evaluation														■	■			
Final report																■	■	■

8 Budget

EAFIT University provides data bases for the literature review, software licenses to implement the computer model and the tutor professor.

9 Intellectual property

According to the internal regulation on intellectual property within EAFIT University, the results of this research practise are product of Camila Mejía and Miguel Tamayo as students, and Juan Carlos Rivera as tutor professor.

In case further products, beside academic articles, that could be generated from this work, the intellectual property distribution related to them will be directed under the current regulation of this matter determined by EAFIT University [13].

10 Acronyms List

- TOP: Team Orienteering Problem.
- OP: Orienteering Problem.
- MILP: Mixed Integer Linear Programming.
- CP: Constraint Programming.
- ALNS: Adaptive Large Neighborhood Search.
- TSP: Travelling Salesman Problem.

– CVRP: Capacitated Vehicle Routing Problem.
– VNS: Variable Neighborhood Search.
– TS: Tabu Search.
– ACO: Ant Colony Optimization.
– LNS: Large Neighborhood Search.
– VRPTW: Vehicle Routing Problem with Time Windows.

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