Proposal Report

Research Practise 2

Adaptation of model selecting criteria for nonlinear time series forecasting

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Problem Formulation

Forecasting problems involve predicting events time periods into the future and it can be implemented in different fields. Most forecasting problems involve the use of time series data [1, 2]. In time series forecasting the interest is to discover, with some margin of error, future values of a signal or a function of time, X_t, based on its past values and considering its randomness or fluctuating properties [3].

Although it is difficult to identify the type of a time series, in practice they are divided into two general groups: linear and nonlinear. From the work by Box and Jenkins [4] in the 70s, it gave way to an important effort in the study and application of construction of linear models by mathematical models that represent autoregressive processes (AR), moving average process (MA), and their combination. Many successful practical experiences have shown that this approach can represent the dynamics of many series of real time, which popularized this class of models both in the field academic and professional [5, 6]. However, it has also been found that many real time series have non-linear behavior [2, 7, 8], for which the Box and Jenkins approximation is insufficient to represent these dynamics [5].

Formally, a model of pth-order of nonlinear time series is defined as [9, 10]

$$X_t = f(F_{t-1}; \phi) + a_t \quad , \tag{1}$$

where f is a known nonlinear function of past X_t's and ϕ is a $p \times 1$ vector of parameters. Let $\{X_t\}$ be a stationary and ergodic time series, with F_t the σ-field generated by $\{X_t, X_{t-1}, \ldots\}$. The function f is assumed to have continuous second order derivatives almost surely. The noise process $\{a_t\}$ is assumed to be independent, with mean zero, variance σ_a^2, and finite fourth order moment. It is further assumed that (1) is invertible or equivalently $\{a_t\}$ is measurable with respect to F_t.

Compared to the linear case, the nonlinear time series have been little explored and theory is not sufficient to uncover nonlinearities [11]. One of the most critical issues is to select the appropriate forecasting nonlinear model [2]. The statistical tool used for the evaluation of the accuracy of a selected models are the models selection criteria, which allow given a set of rival models, select the "best" among them [12].

While there are in the literature several decision criteria, most based on the distance between the actual values and their respective predicted values, they are based on assumptions that sometimes are not satisfied by the data under study, and also have some disadvantages that are not considered in practice (see Table 1).

Recently, it has been shown that a good choice to take advantage of the different criteria is through a weighted average of them. However the proposed ways to calculate the weights have not been entirely successful [8, 13]. In addition, in the literature there are no guidelines on which criteria to use, bearing in mind the inherent behavior of the time series.

In this research project the intention is to propose a weighted criterion for selecting models of nonlinear time series, using statistical techniques that consider the inherent characteristics of the series to determine the weights.

Objectives

General Objective

Formulate a criterion for selecting models of nonlinear time series using multivariate analysis techniques and the inherent characteristics of the series.

Specific Objectives

- Identify the different selection criteria formulated in the literature for nonlinear time series.
- Determine the multivariate analysis techniques that allow the creation of synthetic indicators according to the characteristics of the data.
- Establish a methodological framework that considers the characteristics of the data and consider the advantages of the proposed selection criteria in literature to date.
- Validate the feasibility of the proposed methodology by experimental data.

Preceding Work

Modeling nonlinear time series

While there is no single methodology for modeling phenomena that have only temporary data for forecasting in the time series, they all follow the key steps of specification, estimation, validation and prognosis. Abraham and Ledolter [17] specify that in general the modeling methodology of time series for forecasting consists of two stages: The first stage is the model-building phase, and the second is the forecasting phase (Figure 1).

Model selection criterion	Definition	Disadvantages
SSE	$\sum_{i=1}^{T}(y_i - \hat{y}_i)^2$	It has overfitting problems. It is not invariant to linear scale transformations [14] .
RMSE	$\sqrt{\frac{1}{T}SSE}$	It has over-fitting and consistency problems [13].
AIC	$\log\left(\frac{SSE}{T}\right) + \frac{2m}{T}$	It requires normal data [2]. Poor performance in nonlinear time series [15]. Presents a bad performance in nonlinear time series [14].
AICC	$\log\left(\frac{SSE}{T}\right) + \frac{2m}{T-m-1}$	Presents a bad performance in nonlinear time series [14].
BIC	$\log\left(\frac{SSE}{T}\right) + \frac{m\log(T)}{T}$	The penalty term can become dominant [2].
MAPE	$\frac{1}{T}\sum_{i=1}^{T}\left\lvert\frac{(y_i - \hat{y}_i)}{y_i}\right\rvert$	It has consistency problems [13].
DA	$\frac{1}{T}\sum_{i=1}^{T} a_i$, where $a_1 = \begin{cases} 1 & if(y_{i+1} - y_i)(\hat{y}_{i+1} - y_i) > 0 \\ 0 & otherwise \end{cases}$	Large model penalization [8].
MDA	$\frac{\sum_{i=1}^{T-1} D_i}{T-1}$, where $D_i = (A_i - F_i)^2$	Large model penalization [8].
$\bar{R}^2$	$1 - \frac{SSE/(T-m)}{\sum(y_i - \hat{y}_i)^2/(T-1)}$	Inappropriate measure in the field of nonlinear fitting [16].

Table 1: Disadvantages of model selection criteria, where m is the number of parameters and T the number of observations

Stages to construct a time series model for forecasting [17]

Phase 1: Model-Building Phase

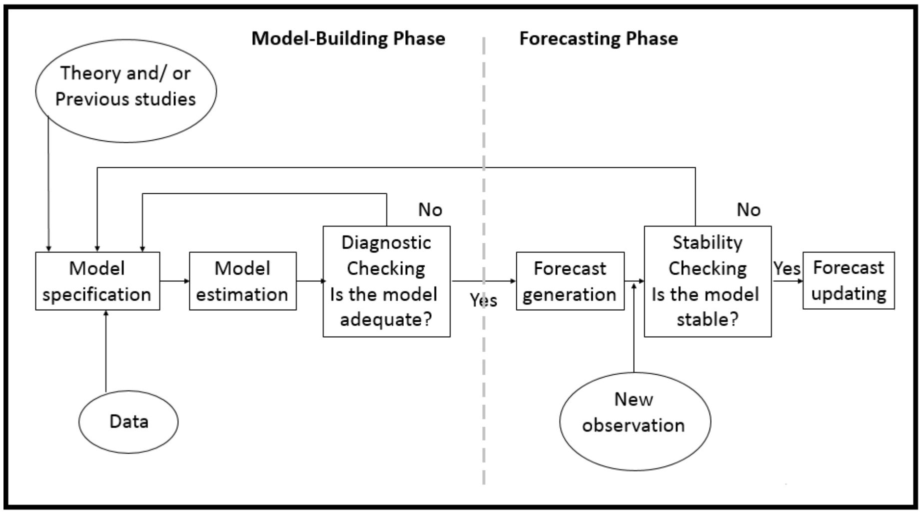

Figure 1: Conceptual framework of a forecasting system. Taken from [17]

A model for forecasting is constructed from measurements of observations and theory (economics, etc.) available. In some cases this theory may suggest certain structures of the model; in other cases, this theory can not exist or be incomplete, and the available data should be used to specify an appropriate model. To choose the structure of forecasting model, the following criteria must be keep:

- The degree of accuracy required.
- The desired forecast horizon.
- The maximum tolerable cost for forecasts.
- The degree of complexity required.
- Data availability.

Moreover, the proposed model generally contains unknown parameters to be estimated in the next step using conventional estimation methods. Finally, it is needed to inspect if the model is appropriate. This should be done to avoid inadequate variables in the model and have an incorrect specification of the functional relationship. If the model is not satisfactory, it must be specified again, and the iterative cycle model specification-estimation-forecasting should be repeated until a satisfactory model is found. This is where the model selection criteria play an important role (Table 1).

Phase 2: Forecasting Phase

At this stage the final model is used for forecasting. The model structure and parameters must remain constant during the forecast period. The stability of the forecasting model can be assessed by checking against new observations. At this point, the forecast error is calculated to detect changes in the model.

Justification

In the modeling of time series selection criteria it is crucial the success of the model-building phase, as said above, to have success in the second phase.

The importance and originality of this research is:

- A solution for the selection of models for nonlinear time series including multivariate techniques would be investigated.
- Heuristic methods have not been successful in the estimation of weights for the combined methods.
- The criteria are applied without validating the assumptions required, leading to bad decisions.
- It would be implemented a method that not only considers the time series characteristics, but the advantages of the existing criteria.

Scope

Validation would be done with experimental (not real) data. A new selection criterion is not going to be proposed, but a methodology that consider other ways to select the weights.

Methodology

In order to ensure a proper development of the project, there will be one hour per week available to be focused in the current research with the tutor plus the time invested on individual study. Initially there will be a search of the state of art databases. Then a study of the necessary contents explored in order to achieve the objectives of the project and then formulate the methodology. Finally the experimental data for validation of the proposed method is generated with the respective analysis. Reports and presentations will be made to document the results obtained throughout the investigation.

Schedule of Activities

It is presented in Table 2 an estimated schedule of the different phases of the project with the respective deadlines. Different documents and presentations must be elaborated according to the Research Practise terms.

Activity	**Start**	**End**
Review of literature	January 30	March 10
Proposal report	January 30	February 12
Oral presentation of the proposal report	February 12	February 19
Identification of the methods	February 19	March 10
Selection of the multivariate technique	March 10	April 1
Oral progress report	April 1	April 8
Method implementation	April 8	April 20
Validation with experimental data	April 20	May 5
Project report	April 10	May 20
Project presentation	May 20	June 7

Table 2: Schedule of Activities.

Intellectual Property

According to [18], Andrea Molina-Alonso and Myladis Rocio Cogollo-Flórez share intellectual property in this research equally.

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