Heuristic and exact solution strategies for the Team Orienteering Problem

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Proposal Presentation

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Team Orienteering Problem (TOP)

The team orienteering problem is a generalization of the orienteering problem (OP), where *m* vehicles are available to visit *n* nodes and the goal is to determine *m* routes, without exceeding given thresholds, that maximize the total collected prize. No node can be visited more than once by one or several routes and there is the possibility of not visiting all nodes [Chao et al., 1996].

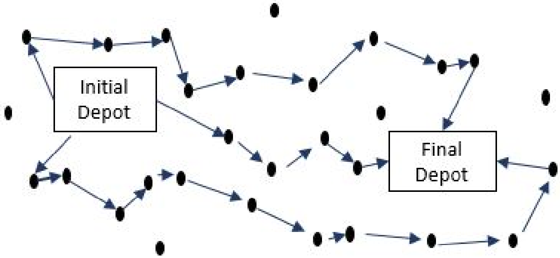

Figure: TOP Example

Mathematical Model

Sets, decision variables and parameters

$$
\begin{aligned}
V &: set\ \ of\ \ all\ \ nodes \\
V' &: set\ \ of\ \ required\ \ nodes\ \ V \setminus \{0, n+1\} \\
V^1 &: V \setminus \{0\} \\
V^2 &: V \setminus \{n+1\} \\
x_{ij}(d.v) &: equal\ \ to\ \ 1\ \ if\ \ arc\ \ (i,j)\ \ is\ \ traversed \\
L_i(d.v) &: traversed\ \ distance\ \ from\ \ node\ \ 0\ \ to\ \ node\ \ i \\
p_i &: profit\ \ of\ \ node\ \ i \\
L_{max} &: maximal\ \ length\ \ tour \\
d_{ij} &: distance\ \ between\ \ nodes\ \ i\ \ and\ \ j \\
s_i &: service\ \ time\ \ at\ \ node\ \ i
\end{aligned}
$$

Mathematical Model

$$\max Z = \sum_{i \in V^2} \sum_{j \in V^1} p_j \cdot x_{ij}$$

$$\sum_{j \in V^2} x_{0j} \leq m$$

$$\sum_{i \in V^1} x_{i,n+1} = \sum_{j \in V^2} x_{0j}$$

$$\sum_{i \in V^2} x_{ij} \leq 1 \qquad \forall\ j \in V^1$$

$$\sum_{j \in V^1} x_{ij} \leq 1 \qquad \forall\ i \in V^2$$

Mathematical Model III

$$\sum_{i \in V^2} x_{ij} = \sum_{i \in V^1} x_{ji} \qquad \forall\, j \in V^1$$

$$L_j \geq L_i + (d_{ij} + s_i) \cdot x_{ij} - L_{max} \cdot (1 - x_{ij}) \qquad \forall\, i \in V^2, j \in V^1$$

$$L_i \leq L_{max} \qquad \forall\, i \in V^2$$

$$x_{ij} \in \{0, 1\} \qquad \forall\, i \in V^1, j \in V^2$$

$$L_i \geq 0 \qquad \forall\, i \in V$$

Additional valid inequalities:

$$L_i + s_i + d_{i,n+1} \leq L_{max} \qquad \forall\, i \in V'$$

$$\sum_{j \in V^1} d_{ij} \cdot x_{ij} \in sos^{1*} \qquad \forall\, i \in V^2$$

$^*sos^1$: special ordered sets type 1

Objectives

- To compare exact solution approaches for TOP based on constraint programming (CP) and mixed integer linear programming (MILP) by using Cplex.
- To propose a heuristic algorithm based on the hybridization of mathematical programming formulations and adaptive large neighborhood search heuristics (ALNS).

Constraint Programming

"The essence of constraint programming is a two-level architecture integrating a constraint and a programming component. The constraint component provides the basic operations of the architecture and consists of a system reasoning about fundamental properties of constraint systems such as satisfiability and entailment. The constraint component is often called the constraint store, by analogy to the memory store of traditional programming languages. Operating around the constraint store is a programming-language component that specifies how to combine the basic operations, often in non-deterministic way" [Gass and Harris, 2012, p 268].

Constraint Programming Example

Matlab:

$a = 10;$
$sum = 0;$
for $i = 1 : a$
    $sum = sum + i;$
end
if $sum == 55$
    $disp('True');$
else
    $disp('False');$
end

Result: True.

CP:

int $a = 1..10$
sum$(x\ in\ a)\ a[x] == 55;$

Result: True.

Matheuristic ALNS

- First proposed by [Pisinger and Ropke, 2007]
- Set of destroy and repair procedures
- Repair procedures will be replaced by solving mathematical models

Questions?

References

Chao, I.-M., Golden, B. L., and Wasil, E. A. (1996).
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