Mathematical Modelling of a Deregulated Electricity Market With Different Production Capacities

Research Practise 2 Proposal Presentation

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February 19th, 2016

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- $d \in \{1, ..., K\}$; random variable, which determines electricity demand for the next day, with probability distribution $\pi_i = Pr(d = i)$.
- $p_i \in [0, \bar{p}]$; unitary price offered by generator firm g_i, and $\bar{p}$ is a regulatory maximum price.

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$$u_i = \delta_\rho(i)(d - K_{\rho-1})(p_{n_\rho} - c_{n_\rho}) + \sum_{m=1}^{\rho-1} \delta_m(i) k_{n_m}(p_{n_\rho} - c_{n_m})$$

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with $\rho = max\{j : K_{j-1} < d\}$ and $\delta_m(i) = 1$ if $i = n_m$ and $\delta_m(i) = 0$, otherwise.

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In order to obtain the expected aggregated supply curve in an equilibrium, we have to work in a Nash equilibrium situation, thus, adapting the definition of Nash equilibrium in finite mixed strategies given by [Navarro et al., 2003], we have that $F_1, F_2, ..., F_N$ is a Nash equilibrium if $\forall n \in \{1, ..., N\}$

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$$E_{F_1,F_2,...,F_n,...,F_N}(u_n) \geq E_{F_1,F_2,...,\tilde{F}_n,...,F_N}(u_n)$$

where $\tilde{F}_n$ is any other possible cumulative distribution function for the price offered by g_n.

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It is known (adapting Theorem 3.1 in [Navarro et al., 2003]) that $F_1, F_2, ..., F_n$ is a Nash equilibrium if and only if for any n, the expected profit $\Phi_n(p)$ of firm g_n given that it plays the pure strategy $p_n = p$ and the other $g_i's$ play $F_i's$, is independent of p.

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Thus, we have to find an explicit formula for $\Phi_n(p)$, which is necessary if we want to obtain the equilibrium strategies $F_1, F_2, ..., F_n$, and once the $F's$ are obtained, is possible to simulate the game many times to obtain an approximation to the expected aggregated supply curve.

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An explicit formula for $\Phi_n(p)$ in the case that all firms are equal is given in Appendix A in [von der Fehr and Harbord, 1993].

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Specific

- Understand each component of the explicit formula of $\Phi_n(p)$ in the case where all firms have the same properties.
- Define the pure strategy space and mixed strategy space for each one of the players.
- Propose a structure of the formula for $\Phi_n(p)$, when $N = 4$ using ideas of the previous case.

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Preceding Research

A first approach to deregulated electricity markets from game theory is given by [Richard J. Green, 1992], who consider the British case as a duopoly, giving some empirical simulations of electricity market, making some observations about a possible equilibrium of the market.

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After that, [von der Fehr and Harbord, 1992, 1993] gave a more theoretical treatment of previous duopoly with some generalizations in multiple firms case.

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More recently, in Colombia, the problem of deregulated electricity markets has been treated from several points of view, like multi-agent systems [Gallego et al., 2008] and artificial intelligence [Quintero et al., 2014].

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Actually, some researchers of EAFIT University are working on this problem. This project is the continuation of the work done in [Cadavid et al., 2016], where we considered a very simple case of this market type.

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Justification

The aim of the research which this project come from is to study expected aggregated supply curves in the cases where generator firms have different production capacities and there is an equilibrium.

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In order to do that, is necessary to make simulations of the auctions, but this is not possible if we do not have the equilibrium strategies played by each firm, and those are obtainable if we know an explicit form of $\Phi_n(p)$.

Now, finding that formula becomes a very complex probability problem if we do not know anything about how could it be, then, looking for that formula in some representative case ($N = 4$) which tells us how the general formula looks, is an useful step for the principal research.

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Scope

This work is an intermediate step for the research of professor Cadavid and his colleagues. It just includes the search of a formula for $\Phi_n(p)$ in the case $N = 4$ to make easier the search of the general formula, which is essential for the general research.

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