

A Methodology for Commodity Trading in Colombia

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Some Definitions

- **Commodity:** Basic good used in commerce that is interchangeable with other commodities of the same type. Most often used as inputs in the production of other goods or services.
- **Trading:** Concept that involves multiple parties participating in the voluntary negotiation and then the exchange of one's goods and services for desired goods and services that someone else possesses.
- **Derivative:** Security with a price that is dependent upon or derived from one or more underlying assets.

Futures and Forwards

- **Future Contract:** Financial contracts obligating the buyer to purchase an asset at a predetermined future date and price.
- **Forward Contract:** customized contract between two parties to buy or sell an asset at a specified price on a future date.

Derivatives Valuation

Let f be the price of a derivative. To calculate its price the following Partial Differential Equation must be solved [Black and Scholes, 1973]:

$$\frac{\partial f}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 f}{\partial S^2} + rS \frac{\partial f}{\partial S} = rf \quad (1)$$

where f is the price of the Option, S is the price of the underlying, σ is the volatility of the underlying and r is the free interest rate risk.

Note

The boundary conditions depends on the Option's dynamic.

Assumptions

- The price of the underlying is a Geometric Brownian movement
- No transaction costs.
- The assets are perfectly divisible.
- The underlying pays no dividends during the life of the option.
- No arbitrage opportunities.
- The negotiation of assets is continuing.
- Free interest rate risk r is constant for all maturities.

Black-76 model

In 1976, Fisher Black [Black, 1976] presented for the first time a variant of the Black-Scholes model which had, as its principal application, a focus for pricing options on future contracts. This model, which will be introduced further, has more light assumptions so it may be applied to both future and forward contracts in a more suitable way.

Black-76: Futures I

The T-futures price $f_{t,T}$ for a given commodity can be explained by Equation 2.

$$df_{t,T} = \sigma f_{t,T} dW_t^d \quad (2)$$

where $f_{t,T}$ is the price of the T-future contract, σ the volatility of the underlying and W_t^d is a Wiener process.

The assumptions of the Black-76 model for future contracts valuation are:

- The T-futures price is perceived as a driftless lognormal process with respect to the domestic risk neutral measure.
- T will be fixed a priori.
- $f_t = f_{t,T}$.

Black-76: Futures II

By applying the Ito's lemma we obtain:

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 f^2 \frac{\partial^2 V}{\partial f^2} - r^d V = 0. \quad (3)$$

Black-76: Forwards I

The T-forward price $F_{t,T}$ for a given commodity can be explained by Equation 4.

$$dF_{t,T} = \sigma F_{t,T} dW_t^d \quad (4)$$

where $f_{t,T}$ is the price of the T-forward contract, σ the volatility of the underlying and W_t^d is a Wiener process. The assumptions of the Black-76 model for forward contracts valuation are:

- The T-forward price is perceived as a driftless lognormal process with respect to the domestic risk neutral measure.
- The interest rates are assumed to be non stochastic.
- $F_t = F_{t,T}$.

Black-76: Forwards II

By applying the Ito's lemma we obtain:

$$\frac{\partial V}{\partial t} + \frac{1}{2} \sigma^2 F^2 \frac{\partial^2 V}{\partial F^2} - r^d V = 0. \quad (5)$$

Framework

- I. Clark considered the Black-Scholes model with or not a non-zero convenience yield [Clark, 2014] holding the same assumptions of the Black-Scholes model.
- Fisher Black [Black, 1976] presented for the first time a variant of the Black-Scholes model which had, as its principal application, a focus for pricing options on future contracts.

Objectives

General Objective:

- Establish a methodology for Commodity Trading in Colombia based on the results obtained from the implementation of Black-76 model to real data taken from a commodity market.

Objectives

Specific Objective:

- Understanding the variations the Black-Scholes model had to become the black-76 model.
- Obtain historical data to propose a structure for the spot price of a specific commodity in the colombian market in order to implement the Black-76 model for futures and forwards contracts.
- Establish the conditions and steps to follow in a Colombian Market to implement a formal commodity trading.

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