Modeling Colombian yields with a macro-factor affine term structure model

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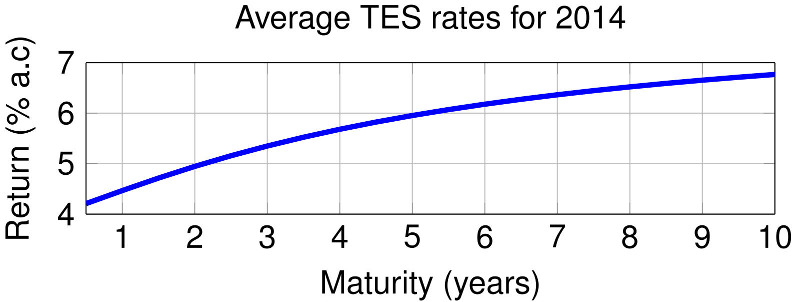
Outline

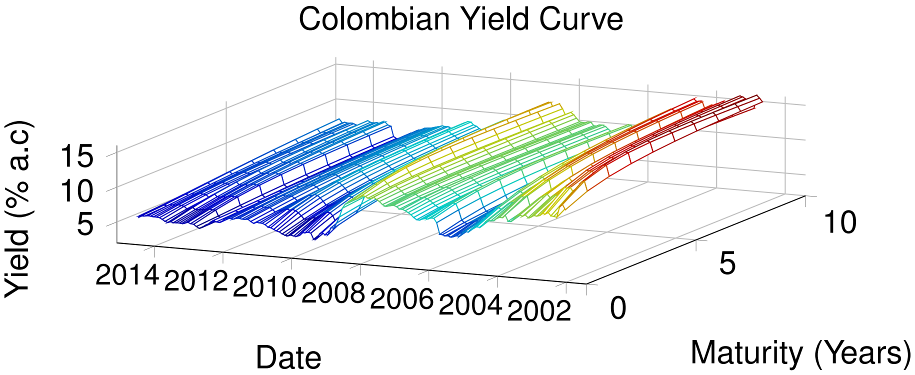
Interest rates

- Vary with *time* (t) and *maturity* (τ).
- Higher maturities (usually) have higher expected returns.



The term structure of interest rates

- Relationship between interest rates and their maturity.
- Dynamic and (mostly) not observable.



Affine term structure models

- Introduced by [Duffie and Kan, 1996].
- Model yields over time and maturity $\gamma_\tau(t)$ as functions of N latent state variables $X(t)$:

$$\gamma_\tau(t) = A(\tau) + B(\tau)^\top X(t)$$

$$r(t) = \lim_{\tau \to 0} \gamma_\tau(t) = \delta_0 + \delta_1^\top X(t)$$

- $X(t)$ captures changes over **time**.
- $A(\tau)$ and $B(\tau)$ change over **maturity**.

The risk neutral measure Q

Q is a probability measure in which prices are expected discounted payoffs:

$$P(X(t), \tau) = E^Q \left[exp \left(- \int_t^{t+\tau} r(u)\, du \right) |X(t) \right]$$

$X(t)$ is assumed to follow an affine diffusion under Q:

$$dX(t) = \mu^Q(X)dt + \sigma(X)dW^Q(t)$$

$W^Q(t)$: N-dimensional Brownian motion.

Price of risk and physical measure P

$\Lambda(X(t)) : \mathbb{R}^3 \to \mathbb{R}^3$ models how agents price risk.

The P dynamics are obtained as:

$$\begin{aligned} dX(t) &= (\mu^Q(X) + \Lambda(X))dt + \sigma(X)dW^P(t) \\ &= \mu^P(X)dt + \sigma(X)W^P(t) \end{aligned}$$

General objective

To model Colombian yields with an ATSM that incorporates macroeconomic factors.

Specific objectives

- Implement an estimation methodology.
- Include macroeconomic variables in the models.
- Compare with [Velásquez-Giraldo and Restrepo-Tobón, 2016].

Outline

Dynamic yield curve models

- 1996 — Duffie & Kan introduce ATSMs.
- 2000 — Dai & Singleton Propose canonical form for ATSMs.
- 2006 — Diebold & Li introduce dynamic Nelson-Siegel models.
- 2010 — Christensen et al. introduce no-arbitrage NS models.
- 2011 — Hamilton & Wu show identification problems in ATSMs.

Dynamic yield curve models in Colombia

- 2008 — Restrepo-Tobón & Botero-Ramírez test the CIR and HW models.
- 2010 — Melo-Velandia & Castro-Lancheros use a dynamic NS model with macro-factors.
- 2014 — Maldonado-Castaño et al. adjust a dynamic NS model.
- 2016 — Velásquez-Giraldo & Restrepo-Tobón test various ATSMs.

Previous Research Practise

[Velásquez-Giraldo and Restrepo-Tobón, 2016] tested various ATSMs with Colombian data and found:

- Estimation is very complex.
- Three-factor Gaussian model worked best.
- Latent factors retain interpretation.
- Shorter yields harder to forecasts.

Outline

On a local level

- Create literature on no-arbitrage models.
- Macro-factor ATSMs haven't been tested yet.
- Study interesting unobservables (short rate & risk price).
- Publish forecasting benchmarks.
- Learn about the yield curve's reactions to the economic environment.

On an international level

- Analyze performance of ATSMs with daily data.
- Possibly contribute to the estimation & identification literature.
- Include daily economic time series in the models.

Outline

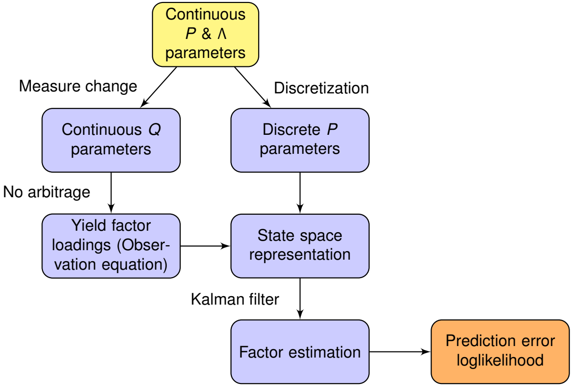
Data

Yields Zero coupon yields from Bloomberg with maturities from 0 to 10 years.

Macro-factors Not defined yet. Also taken from Bloomberg.

Time period 2005-2015.

Preliminary estimation procedure



Based on [Christensen et al., 2011].

After estimation

- Modify the baseline model with macro-factors.
- Analyze fit and forecasting performances.
- Compare results with [Velásquez-Giraldo and Restrepo-Tobón, 2016].
- Interpret results.

Outline

1. Problem formulation
2. Preceding research
3. Justification
4. Methodology
5. References

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Thanks for your attention!