Mathematical Modelling of a Deregulated Electricity Market With Different Production Capacities

Research Practise 2 Progress Presentation

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Outline

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- $p_i \in [0, \bar{p}]$; unitary price offered by generator firm g_i, and $\bar{p}$ is a regulatory maximum price.

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with $\rho = max\{j : K_{j-1} < d\}$ and $\delta_m(i) = 1$ if $i = n_m$ and $\delta_m(i) = 0$, otherwise.

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In order to obtain the expected aggregated supply curve in an equilibrium, we have to work in a Nash equilibrium situation, thus, adapting the definition of Nash equilibrium in finite mixed strategies given by [Navarro et al., 2003], we have that $F_1, F_2, ..., F_N$ is a Nash equilibrium if $\forall n \in \{1, ..., N\}$

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$$E_{F_1,F_2,...,F_n,...,F_N}(u_n) \geq E_{F_1,F_2,...,\tilde{F}_n,...,F_N}(u_n)$$

where $\tilde{F}_n$ is any other possible cumulative distribution function for the price offered by g_n.

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It is known (adapting Theorem 3.1 in [Navarro et al., 2003]) that $F_1, F_2, ..., F_n$ is a Nash equilibrium if and only if for any n, the expected profit $\Phi_n(p)$ of firm g_n given that it plays the pure strategy $p_n = p$ and the other $g_i's$ play $F_i's$, is independent of p.

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Thus, we have to find an explicit formula for $\Phi_n(p)$, which is necessary if we want to obtain the equilibrium strategies $F_1, F_2, ..., F_n$, and once the $F's$ are obtained, is possible to simulate the game many times to obtain an approximation to the expected aggregated supply curve.

Problem's Definition

An explicit formula for $\Phi_n(p)$ in the case that all firms are equal, is given in Appendix A in [von der Fehr and Harbord, 1993], as follows:

$$\Phi_n(p) = \sum_{i=1}^{N} \pi_i \{Pr[p_{n_{i-1}} \leq p \leq p_{n_{i+1}} | p_n = p]p + \int_p^{\bar{p}} \rho dF_{n_i}(\rho)\}$$

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Specific

- Understand each component of the explicit formula of $\Phi_n(p)$ in the case where all firms have the same properties.
- Define the pure strategy space and mixed strategy space for each one of the players.
- Propose a structure of the formula for $\Phi_n(p)$, when $N = 4$ using ideas of the previous case.

Outline

We noted that the pure strategy space for the player g_i is $S_i = [0, \bar{p}]$, and the mixed strategy space is

$$\Sigma_i = \{F : [0, \bar{p}] \to [0, 1] \mid F \text{ is a cumulative distribution function}\}$$

Now, if we consider that there could be ties between prices offered, the coordinator becomes another player, and its pure strategy space is given by

$$S_c = \{R : [0, \bar{p}]^N \to \{1, 2, ..., N\}^2 \mid R(p_1, ..., p_N) \text{ is a ranking of lowest prices}\}$$

In order to get $E[u_1|p_1 = p]$, we considered a partition of the sample space given by the events
$\varepsilon(B, \{c\}, A) := p_b < p_c < p_a, \forall_{b \in B}, \forall_{a \in A} \quad \wedge \quad K_B < d \leq K_B + k_c$
where $K_B = \sum_{b \in B} k_b$ and $B \cupdot \{c\} \cupdot A = \{1, 2\}$.

Results

$\Phi_n(p)$ When N=2

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$$E[u_1|p_1 = p] = (p - c_1)(1 - F_2(p)) \sum_{i=1}^{k_1} i\pi_i + (p - c_1)F_2(p) \sum_{i=1}^{k_1} i\pi_{k_2+i}$$
$$+ \left(\int_{(p,\bar{p})} k_2(\rho - c_1)G'(\rho)d\rho + (G(\bar{p}) - G(\bar{p}^-))(\bar{p} - c_1)k_1 \right) (1 - F_2(p)) \sum_{i=1}^{k_2} \pi_{k_1+i}$$

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where

$$G(\rho) = Pr[p_2 \leq \rho | p_1 < p_2, p_1 = p] = \frac{F_2(\rho) - F_2(p)}{1 - F_2(p)}$$

and

$$G(\bar{p}^-) = \lim_{\rho \to \bar{p}^-} G(\rho)$$

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Further Work

Using the previous idea, we will try to find $\Phi_n(p)$, in the cases $N = 3$ and $N = 4$, which could lead us to the formula of $\Phi_n(p)$ for any $N \in \mathbb{N}$. If we obtain that general formula, we hope to continue this research with a computacional implementation.

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Bibliography

[Navarro et al., 2003] Navarro, J., Tena, E., and Pastor, J. (2003).
Teoría de Juegos.
Out of Series. Editorial Alhambra S. A. (SP).

[von der Fehr and Harbord, 1993] von der Fehr, N.-H. M. and Harbord, D. (1993).
Spot market competition in the uk electricity industry.
University of Oslo.