Study and application of an Asset-Liability Management model for a life insurance product

Francisco González-Piedrahita and Sebastián Rincón-Montoya
Mathematical Engineering
EAFIT University
fgonzal6@eafit.edu.co - *srincon2@eafit.edu.co*

Ledwing Osorio-Cárdenas
Suramericana S.A.
lgosorio@sura.com.co

Francisco Zuluaga-Díaz
EAFIT University
fzuluag2@eafit.edu.co

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Abstract

In this article, an Asset Liability Management model is implemented for a life insurance product, taking into account its fundamental principles, such as claims, withdrawals and primes. It was built an optimal portfolio applying Markowitz theory and the estimation for the COLCAP index with an artificial neural network approach as a decision rule to invest in shares. To estimate the obligations of the company or liabilities, it was used a Lee-Carter model, which projects the claims and withdrawals behavior.

1 Introduction

When insurance companies offer policies that protect an individual or business of any risk they assume liabilities with these clients. Companies' assets suffer transformations which are usually related to customers needs. This project proposes different approaches to manage the risk associated with life insurance products.

In Finance and Economy, 'solvency' is the word used to express the stage where a company exceeds his liabilities with their current assets. Solvency is also described as an indicator or a relation that represents, for each monetary unit of debt, how many monetary units the company haves for deal with his debts [1]. Therefore, to calculate the solvency that a life insurance product needs to support the portfolio at any moment, is quite important to achieve the stability the business wants.

Solvency problem is a concern to all insurance companies, this is why the European Union established a set of rules and practices that guarantee the right measurement of the claims update named Solvency I [1]. With the evolution of the insurance market, Solvency I became outdated in the appropriate regulation, so the European parliament declare a new directive for the insurance market called Solvency II [2]. The key objectives of this regulation are the consumer protection, modernize the supervision and increased international competitiveness. This new set of practices were founded around three pillar, these are, the financial requirement that involves Solvency I, the governance and supervision that specify the effective risk management system and the reporting and disclosure, this is oriented to make the insurers reveal valuable information around the risk facing and the transparency that the published information must have [3].

All this framework of solvency propose an environment of development for the guild of insurers that seeks a uniform evolution of the business that provides an optimal performance of the assets, in order to achieve this objective is important to make a tracing for financial behavior of the company products, this is why is necessary to apply the theory of sufficiency, to know if the products accomplish the requirements of Solvency II and if not, evaluate other alternatives that produce the fit between assets and liability [3].

Nowadays, Superintendence of Finance establishes an Asset Sufficiency Test to guarantee that insurance companies will respond to their obligations. However, this methodology is not enough for representing all the assets management that the product requires. It is then necessary to implement an Asset-Liability Management (ALM) model to administrate policies and procedures that add financial risks associated with changing interest rates, trying to anticipate possible changes in the portfolio [4]. ALM model gives the complement to the sufficiency test, using techniques for coordinating the management of assets and liabilities, expecting that an adequate return may be earned.

The article is structured as it follows: Section II introduces the objective and problem definition of the research, Section III describes the methodology implemented to solve the problem, Section IV contains the most relevant results obtained, which are later explained and analyzed in Section IV; and finally the last section presents the conclusions of the proposed methodology.

2 General and Specific Objectives

General: To find the optimal point where the assets and liabilities fit, avoiding this way, the overruns of the reserves, releasing the excess of retained capital and, by this way, providing the company more resources to invest.

Specifics:

- To analyze different risks tied to asset and liabilities in life insurance products.
- To forecast mortality and estimate its obligations with clients.
- To create an alternative portfolio with reinvestment strategies.

3 Preceding Research

After defining the research project, it is necessary to understand different approaches with several methodologies which can be useful to accomplish the objectives mentioned above. Nowadays these topics are being increasingly used not only by the insurance market but any company that must manage risks subject to mismatches of assets and liabilities. However, as expected, these organizations keep their information confidential and it can become a barrier.

AST methodologies have been lately used by life insurance companies in Chile in order to establish a financial balance. This market has grown meaningfully on the past few decades, becoming one-fifth of the economy of the country, which means an obligation to systematically supervise organizations solvency, which can be seen in [5]. On the one hand, there have been approaches using discrete time stochastic ALM models in life insurance products, one of them describes the most significant characteristics of these products in order to simulate its balance sheets [4], and the other one optimizes by Monte-Carlo the pricing of the embedded options in life insurance contracts [6]. On the other hand, there is another approach using Mortgage Backed Securities (MBS) as an asset and testing the cash flow as an ALM model in order to guarantee that MBS supports an amount of liabilities [7].

ALM models can also be applied demonstrating how solvency elements make a huge impact on the capital requirements, also showing that reducing short-term risk the long term expected returns may also decrease [8]. Because of solvency implementation, objections were not long in coming, proposing to assess the costs of longevity risk management using indemnity based longevity swaps, its cost should be lower than the capital required under solvency regulations [9].

In [10] there is a development for an optimal portfolio in an insurance company based on the theory proposed by Markowitz in [11]. In this article there is an approach for different scenarios and products, such as fire, life, car, accident and burglary. It can be seen some interesting theoretical results but some serious practical limitations.

4 Justification

One of the main concerns of insurance companies is to assess the financial viability and long term stability. This is why they are forced to establish a solvency structure to accomplish these goals. It requires a coherent and comprehensive vision of the risk factors to which a person is exposed, as well as the possibility this risk may materialize.

In Colombia since 21 century there has been an arrival increase from multinational companies which are global leaders in the market, such as, Metlife, MAPFRE, AXA, AIG and so on. Even though Suramericana S.A. has been leader in the life insurance market for more than four decades, it simplifies more competition which also means lower premiums, forcing an improvement in management of their assets, liabilities and risks.

There is another concern for these companies due to regulations made by the Superintendence of Finance. They impose the given rates that must be charged for life insurance products. These rates are the result of the development of a life table with the experience of all insurance companies. In 2017 there will be a new table where it will be seen how mortality has decreased. This is one of the justifications for why they should be more precisely between the obligations with customers and the assets of companies.

5 Methodology

5.1 Data

Data for obtaining optimal portfolio of assets COLCAP, were obtained from the financial software Bloomberg L.P; the closing price of the last 805 days in the stock market was obtained. The IPC and COLCAP values were obtained from the Republic Bank online database. All the data presented below has been normalized to attribute the same importance to all collected data.

There is an additional data which is needed to estimate mortality and will be essential to project or forecast liabilities. This information is taken from a website called: Human Mortality Database [12]. It offers detail information of populations and mortality data for 38 countries or areas around the world. For research purpose we will analyze data from USA given its wide database, specifically death rates by sex and age from 1933 to 2013.

5.2 Markowitz Model

The procedure of Markowitz is a model which constructed an optimal investment portfolio that minimize the volatility. This model is executed in two stages, the first is market exploration, obtaining the required data as performance and expected variation of assets, in the second instance, the assets that will be part of the portfolio are chosen, this stage is executed by picking the six assets with the bigger performance and lower deviation [11]. Markowitz define the participation that each asset may have into the portfolio. The measure of performance is basically:

$$r_t = ln(\frac{p_t}{p_{t-1}}), \tag{1}$$

where r_t is the performance and p_t is the price at time t.

The objective function is to minimize the following risk measure

$$\sigma_p = \sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j Cov(i,j), \tag{2}$$

where w_i is the weigth of the share i on the portfolio and $Cov(i,j)$ is the covariance between the share i and j.

To define a constraint that would regulate the possible risk that the portfolio could take, the cash value at risk is calculated, it is a measure of the amount of potential loss that could happen in an investment or a portfolio of investments over a given time period. This formula is the next one:

$$CVaR_{\alpha} = Z_x^{-1,}(\alpha) * \sqrt{t} * \sum_{i=1}^{n} Inv_i * \sigma_i \qquad (3)$$

where Z_x^{-1} is the inverse standard normal distribution, α is the uncertainty percentage, x is the certainty percentage, t is the exposure period (days), Inv_i and σ_i correspond to the invested capital and the volatility of the share i, respectively. The constraint added to the mMrkowitz model must be:

$$CVaR_{\alpha} < P_k * 10\% \qquad (4)$$

Where P_k is the earned premium at the year k and the 10% correspond to the utility associated to Plan vida Personal and it is the maximum amount that Suramericana is willing to lose.

5.3 Neural Network Experiment

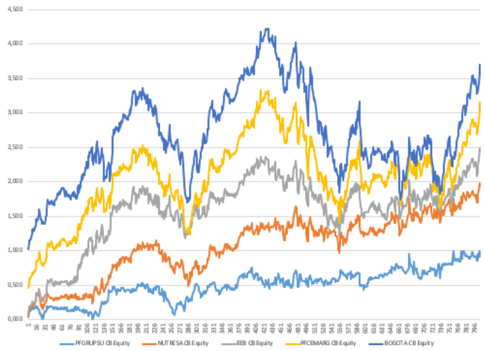
A feed forward neural network was selected for this study, the creation and training of the neural net was made with multiple combinations of the parameters in Table 1.

Table 1: Parameters values

Parameter	Value
Learning rate	[0.1 to 1] step 0.2
Momentum	[0.1 to 1] step 0.1
Layers	[1 to 10] step 1
Perceptron's	[2 to 10] step 1

The inputs selected for the model are the daily prices of five from the six assets used before in the Markowitz model for build the portfolio and another input is the IPC (consumer's price index). The shares prices along the study are shown in the stacked graph in Figure 1.

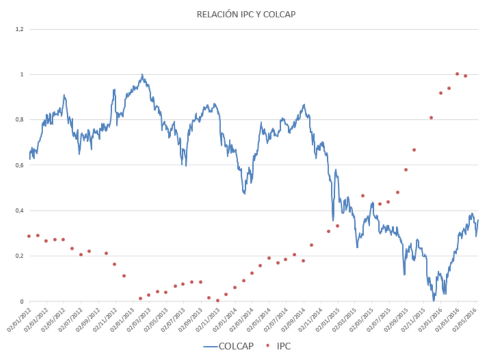
Figure 1: Shares evolution



As can be seen in Figure 1, one of the mainly properties of these shares, is that the price at the end of the study period is higher than the initial price.

The output that the network will learn is the COLCAP index. As can be seen in Figure 2 the IPC growth has a negative effect in the Colombian market, more specifically, on the COLCAP index, hereby.

Figure 2: Colcap and IPC behavior.



In order to know what is the best way to train neural networks for learning in index estimation, we make an algorithm that allows us to train different neural network from changing the settings of its parameters by increasing the value of these at a fixed rate, so that we can observe which of these values the neural network provides the best performance.

5.4 Back Propagation Algorithm

In 1986 the back propagation algorithm was developed boosting the growth of the artificial intelligence field [13]. In fact, back propagation is the most popular learning algorithm to train a set of multilayer perceptrons, understood as a feed forward neural network that maps sets of input data onto a set of appropriate outputs.

The algorithm has the objective of minimizing the error (mean squared error) between estimated and real output; to achieve the objective the optimal connection weights must be found.

5.5 Lee-Carter Model

This model was proposed by Lee and Carter in 1992 [14], pretends to estimate and forecast mortality using time series. It is based firmly on persistent long-term historical patterns and trends, and it provides probabilistic confidence regions for its forecast.

To estimate the model for a given set of time series or matrix of rates $m_{x,t}$, mortality rate for age x and year t, which is defined as the the ratio between the number of deaths $D(x,t)$ and the exposure $E(x,t)$ obtained as the number of people living during the year t. The next equation as a linear combination:

$$ln(m_{x,t}) = a_x + b_x k_t + \varepsilon_{x,t}, \qquad (5)$$

where a_x is the shape of mortality, k_t is the time trend and b_x effect of time at each age x.

Additionally, k_t is is extrapolated using ARIMA time series models. Lee and Carter used a random walk with drift model. The model is, then:

$$k_t = k_{t-1} + d + \epsilon_t, \qquad (6)$$

where d is the average anual change in k_t and ϵ_t is the uncorrelated error.

5.6 ALM Model

Finally, the ALM model is defined as a recursive method with an annual constant change, taking into account projections values for claims and primes and the investment strategies based on Markowitz portfolio:

$$S_t = S_{t-1} + P_{t-1} - C_{t-1} - W_{t-1} - I_t + RI_{t-1}, \qquad (7)$$

where S_t, the total asset amount, P_t, expected prime, C_t, expected claims, I_t, investment amount, W_t, withdrawals and RI_t, return of investment.

Note that the investment amount is calculated a year before is used, it means that the investor cannot used that amount during the present year, hereby the investment must be written as follows.

$$I_t = M_{t-1} + AAA_t, \qquad (8)$$

where M_t, the amount intended to equity portfolio, AAA_t, a triple bond rating which value after the maturation period be equal to the claims in t+1 (C_{t+1}). Thus, the value of the claims of the next year is always assured. The final model is:

$$S_t = S_{t-1} + P_{t-1} - W_{t-1} - M_{t-1} + RI_{t-1} \qquad (9)$$

It is important to notice that the AST is calculated indirectly, because if at any t of the projection, S_t appears as a negative value, the assets are not being enough for support the loss ratio of that product.

6 Results

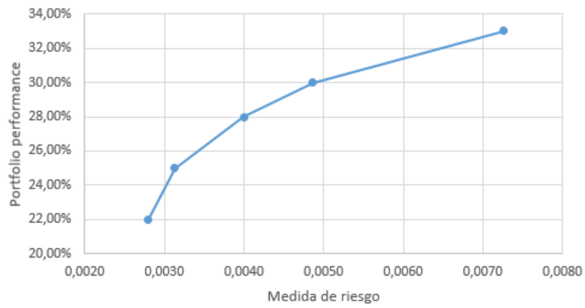
6.1 Markowitz Model

With the Markowitz model, was built the efficiency frontier, which represents the dynamic between the portfolio performance and the risk associated to the investment. In this case were taken into account, five different portfolio participation, this investment structures are shown in Table 2.

Table 2: Investment structures

	Inv 1	Inv 2	Inv 3	Inv 4	Inv 5
Grupsu	40%	28%	19%	14%	13%
Nutresa	20%	31%	22%	17%	16%
EEB	40%	29%	20%	14%	14%
Cemarg	0%	8%	13%	15%	16%
BBogota	0%	3%	14%	20%	21%
Corficol	0%	2%	13%	19%	20%

Figure 3: Markowitz efficient frontier.



The Markowitz quadratic optimization program, was made up in Excel software using the Solver tool.

6.2 Neural Network

After the test described before, the five best architectures were selected according to the lower estimation errors, these are presented in Table 3.

Table 3: Experiment Results 1

Hidden Layers	Neurons	Learning rate	Momentum	Error
5	2	0.7	0.2	0.0667
4	3	0.7	0.1	0.0668
5	2	0.3	0.6	0.0671
5	2	0.5	0.3	0.0675
4	2	0.3	0.9	0.0678
3	2	0.7	0.4	0.0679

Then, the best trained neural network, was used for estimate the COLCAP since the year 2012, the results are shown below in Figure 3. The error of each measure is presented in Figure 4.

Figure 4: COLCAP estimation.

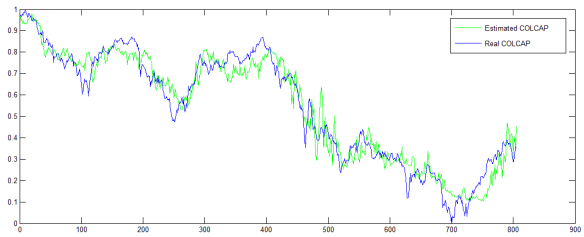
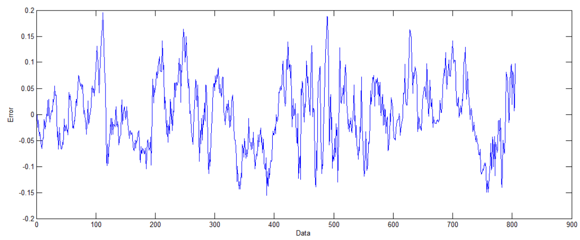


Figure 5: Estimation error



The root mean square error of the estimation is 0.43%, taking into account that the data is normalized.

6.3 Mortality Estimation

In this section Lee-Carter model was applied to obtain mortality rates until 2060, these results are shown in Figures 6 and 7 and are useful to project how claims, withdrawals and primes will behave.

Figure 6: Mortality Estimation

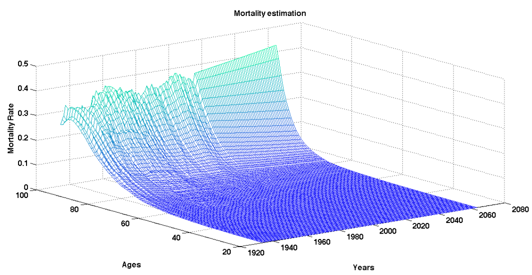
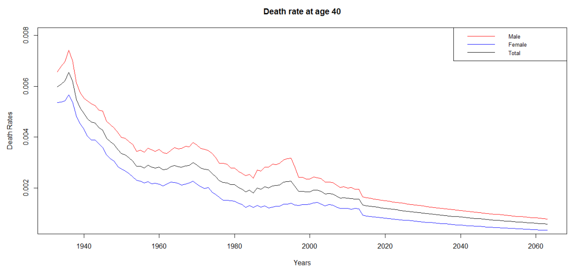


Figure 7: Mortality estimation at age 40



7 Intellectual property

Given that the authors of the project are related to the EAFIT University and Suramericana S.A., the developments and advances during the project execution will be adjusted at the intellectual property regulation from the University and Suramericana. The first one establishes a confidentially and nondisclosure agreement of the information used in any academic project [13].

8 Conclusions

It is important to know the Markowitz efficient frontier, especially in cases where the portfolio calculation is consistently, as in the ALM model, because it allows to define at any instant of time, a portfolio that fits the momentane risk that the investor is willing to face. When the investment structures are created, is necessary that the CVaR is included into the Markowitz model as a severe constraint, as is shown before with a percentage for shield the investment of unexpected events.

Although that with the Colcap index estimation, the obtained results were favorable r, it is important to note that to make predictions with this model share prices must be project and the same with the IPC, providing an exercise which is not trivial and requires additional developments

Mortality estimation allows to increase the amount of investment and reinvestment through the years given that these rates tend to decrease almost in all ages, or at least in the central ages.

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