Study and application of an Asset-Liability Management model for a life insurance product

Project presentation
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Francisco González-Piedrahíta

Sebastián Rincón-Montoya

Advisors:
Ledwing Osorio-Cárdenas - Suramericana S.A.
Francisco Iván Zuluaga Díaz - Department of Mathematical Sciences

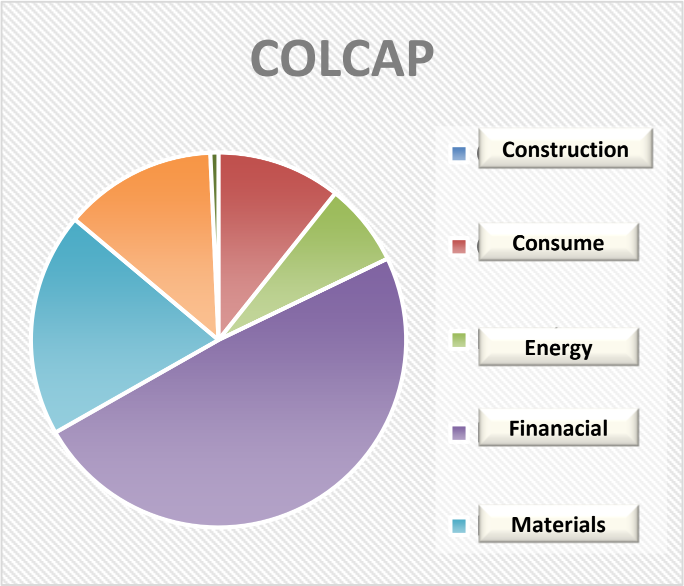
UNIVERSIDAD EAFIT®

Problem Statement

When insurance companies offer policies that protect an individual or business of any risk they assume liabilities with these clients.

The assets of these companies suffer transformations, which are usually related to customers needs. This project proposes an approach to manage the risk associated with life insurance products.

Methodology



Markowitz Model

$$Minimize \quad \sigma_p^2 = \sum_{j=1}^{n}\sum_{i=1}^{n} w_i\, w_j\, Cov(i,j)$$

$$subject\ to \quad \sum_{i=1}^{n} w_i = 1$$

$$\sum_{i=1}^{n} w_i\, \overline{r_i} \geq r_p$$

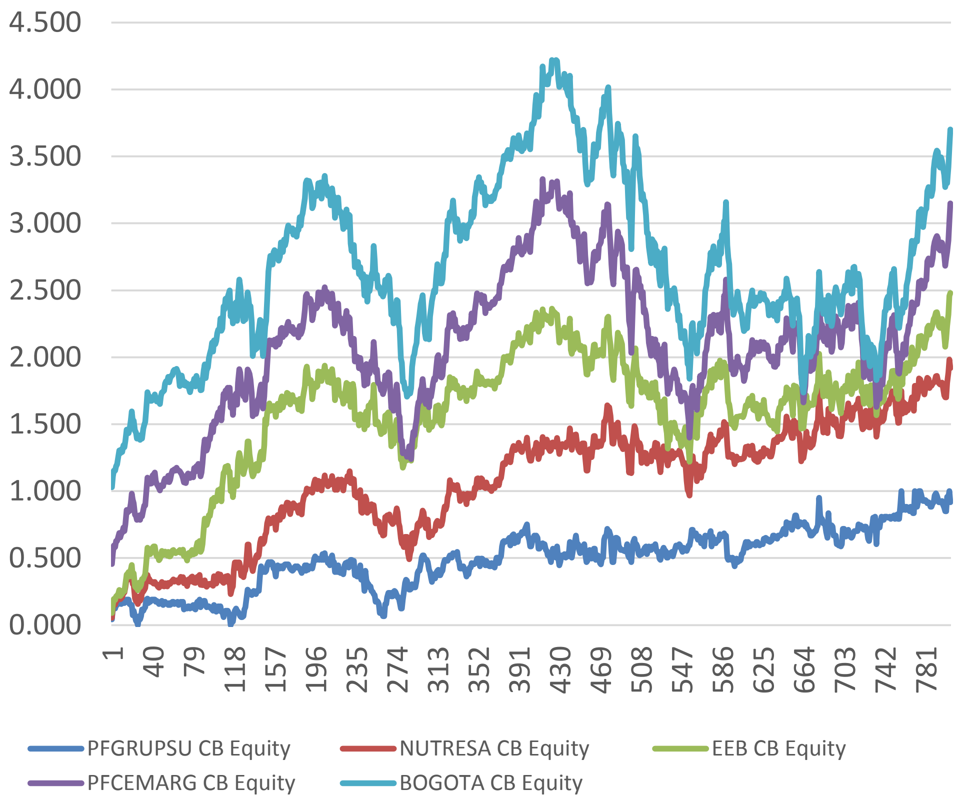
$$w_i > 0$$

$$w_i \leq Mp$$

$$\boxed{VaR_\alpha = Z_x^{-1}(\alpha) * Inv * \sigma * \sqrt{t}}$$

Taken from (Markowitz, 1952).

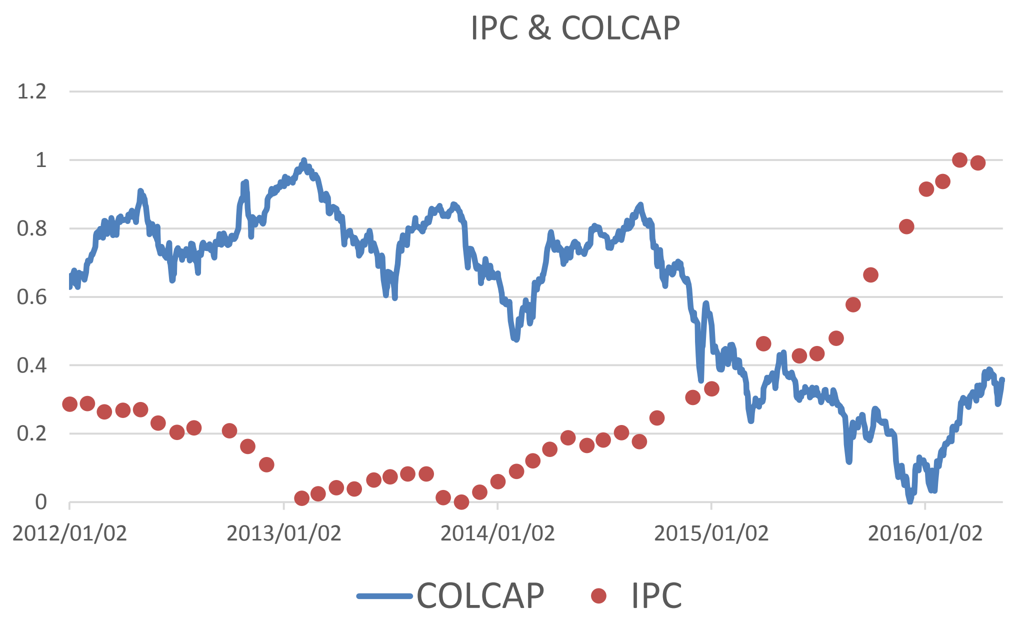
Methodology



	Field	Growth
PFGRUPSU CB Equity	Financial''	58%
NUTRESA CB Equity	Consume	26%
EEB CB Equity	Energy	53%
PFCEMARG CB Equity	Materials	133%
BOGOTA CB Equity	Financial	94%
CORFICOL CB Equity	Financial'	35%

Methodology

Why to use Artificial Neural Network?

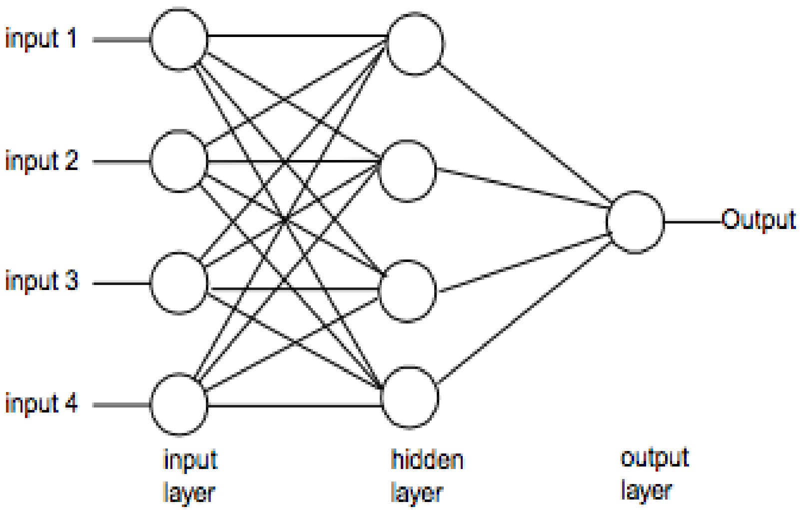


Methodology

Artificial Neural Network

- BP: forward phase, propagate a pattern until the output, then an error is calculated and backward propagated and the weights of the inputs are recalculated (Rumelhart, et al..

- The creation and training of the neural net was made with multiple combinations of the parameters



Parameter	Value
Learning rate	[0.1 to 1] step 0.2
Momentum	[0.1 to 1] step 0.1
Layers	[1 to 10] step 1
Perceptron’s	[2 to 10] step 1

Methodology

Insurance is more commonly purchased by periodic premium payments. The first one is paid at the interception of the agreement and subsequent premiums depend on the ***contingency of insured's survival***. The amount of these premiums required needed is described as it follows:

$$P \cdot \ddot{a} = A,$$

where P is the net premium on an anual basis, $\ddot{a}$ the present value of the appropiate ***life annuity***, and A the net single premium for the insurance as it is proposed in (Wallace, 1991).

Methodology

When a company offers an annuity or an insurance policy, the insurer assumes the obligation of providing certain sums in the future, ***benefit payments***. The insured makes certain payments that its present value are exactly equal to the present value of the payments the insurer will make.

For illustration, we consider an ordinary life policy issued at age x. At the end of t years the present value of the benefit payments is A_{x+t} and the present value of the future net premiums is $P_x \cdot \ddot{a}_{x+t}$. The difference between these amount we consider it as the ***net level premium reserve***:

$$A_{x+t} - P_x \cdot \ddot{a}_{x+t}$$

Methodology

In order to represent mortality through time we chose a model proposed by (Lee and Carter, 1992), where they forecasted on the central mortality rates $m_{x,t}$ for age x and in year t, which is the ratio between the number of deaths $D(x,t)$ and the exposure $E(x,t)$ obtained as the number of people living during the year t.

Describing $m_{x,t}$ as a linear combination of parameters, as following:

$$\ln m_{x,t} = a_x + b_x k_t + \varepsilon_{x,t},$$

where, a_x: shape of mortality, k_t: time trend and b_x: effect of time at each age.

Methodology

Constraints are imposed to obtain a unique solution: the a_x are set equal to the means of $\ln m_{x,t}$ over all time. Also,

$$\sum b_x = 1,$$

$$\sum k_t = 0.$$

The adjusted k_t is extrapolated using ARIMA time series models. Lee and Carter used a random walk with drift model. The model is:

$$k_t = k_{t-1} + d + e_t,$$

where, d: average anual change in k_t and e_t: uncorrelated errors.

Methodology

The ALM model proposed is described as a recursive method with an annual constant change, taking into account projections values for claims and primes and the investment strategies based on Markowitz portfolio.

$$S_t = S_{t-1} + P_{t-1} - C_{t-1} - W_{t-1} - I_t + RI_{t-1} - E_{t-1}$$

where S_t, the total asset amount, P_t, expected prime, C_t, expected claims, I_t, investment amount, W_t, withdrawals, RI_t, return of investment and E_t expenses. The investment must be written as follows.

$$I_t = M_{t-1} + AAA_t \, ,$$

where M_t, the amount intended to equity portfolio, AAA_t, a triple bond rating which value after the maturation period be equal to the claims in $t+1$.

Methodology

The value of the claims of the next year is always assured, thus the final model is:

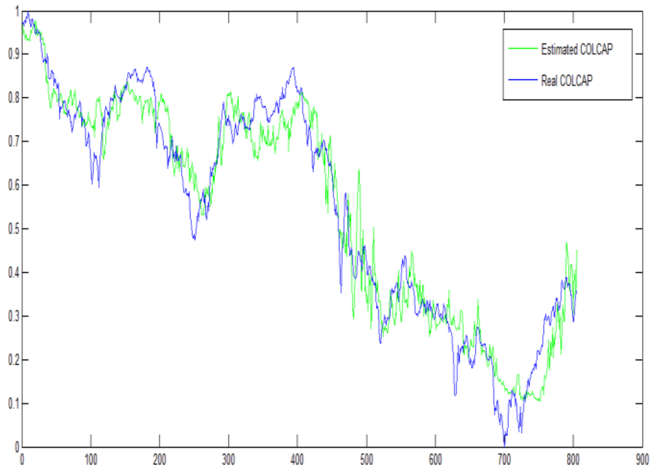
$$S_t = S_{t-1} + P_{t-1} - W_{t-1} - M_{t-1} + RI_{t-1} - E_{t-1}$$

It is important to notice that the AST is calculated indirectly, because if at any t of the projection, S_t appears as a negative value, the assets are not being enough for support the loss ratio of that product.

Results

COLCAP estimation

Hidden layers	*Neurons*	*Learning rate*	*Momentum*	*Error*
5	2	0,7	0,2	0,0667
4	3	0,7	0,1	0,0668
5	2	0,3	0,6	0,0671
5	2	0,5	0,3	0,0675
4	2	0,3	0,9	0,0678
3	2	0,7	0,4	0,0679



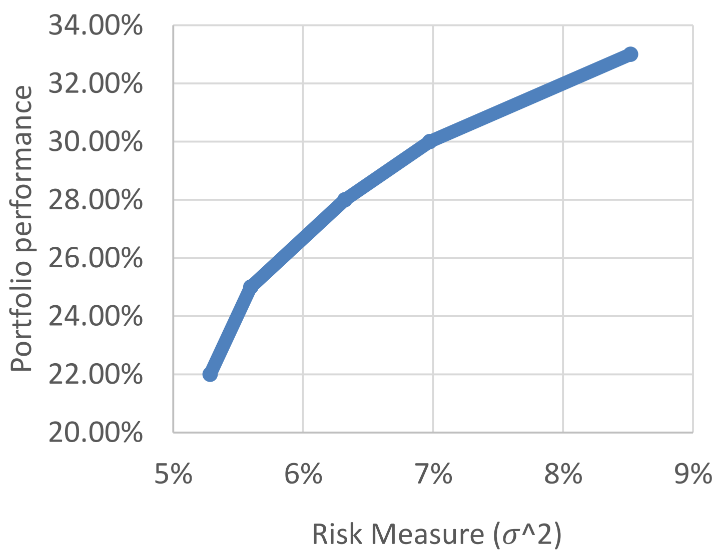
$$Root\ mean\ square\ error = 0{,}43\%$$

Results

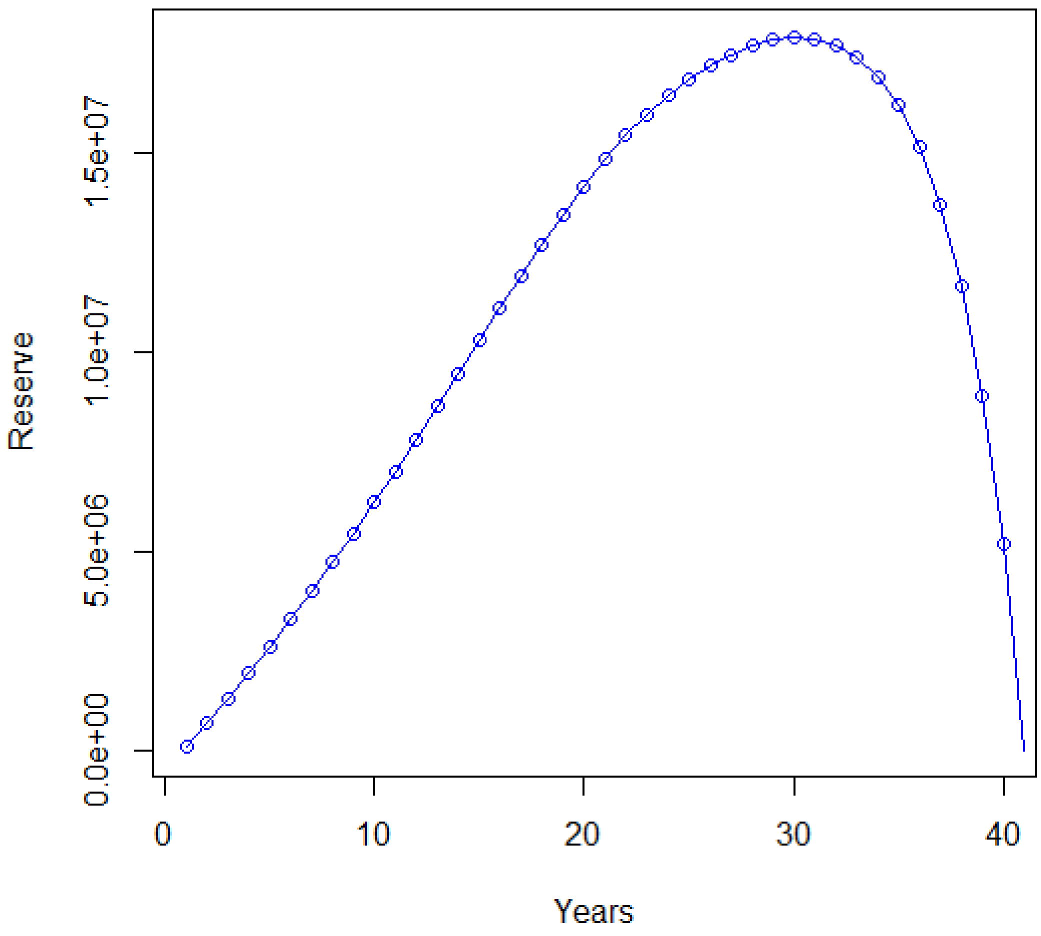
	Scenary 1	Scenary 2	Scenary 3	Scenary 4	Scenary 5
GRUPO SURA	40%	28%	19%	14%	13%
NUTRESA	20%	31%	22%	17%	16%
EEB	40%	29%	20%	14%	14%
ARGOS	0%	8%	13%	15%	16%
B. BOGOTA	0%	3%	14%	20%	21%
B. CORFICOL	0%	2%	13%	19%	20%
Performance	33%	30%	28%	25%	22%

Risky portfolio

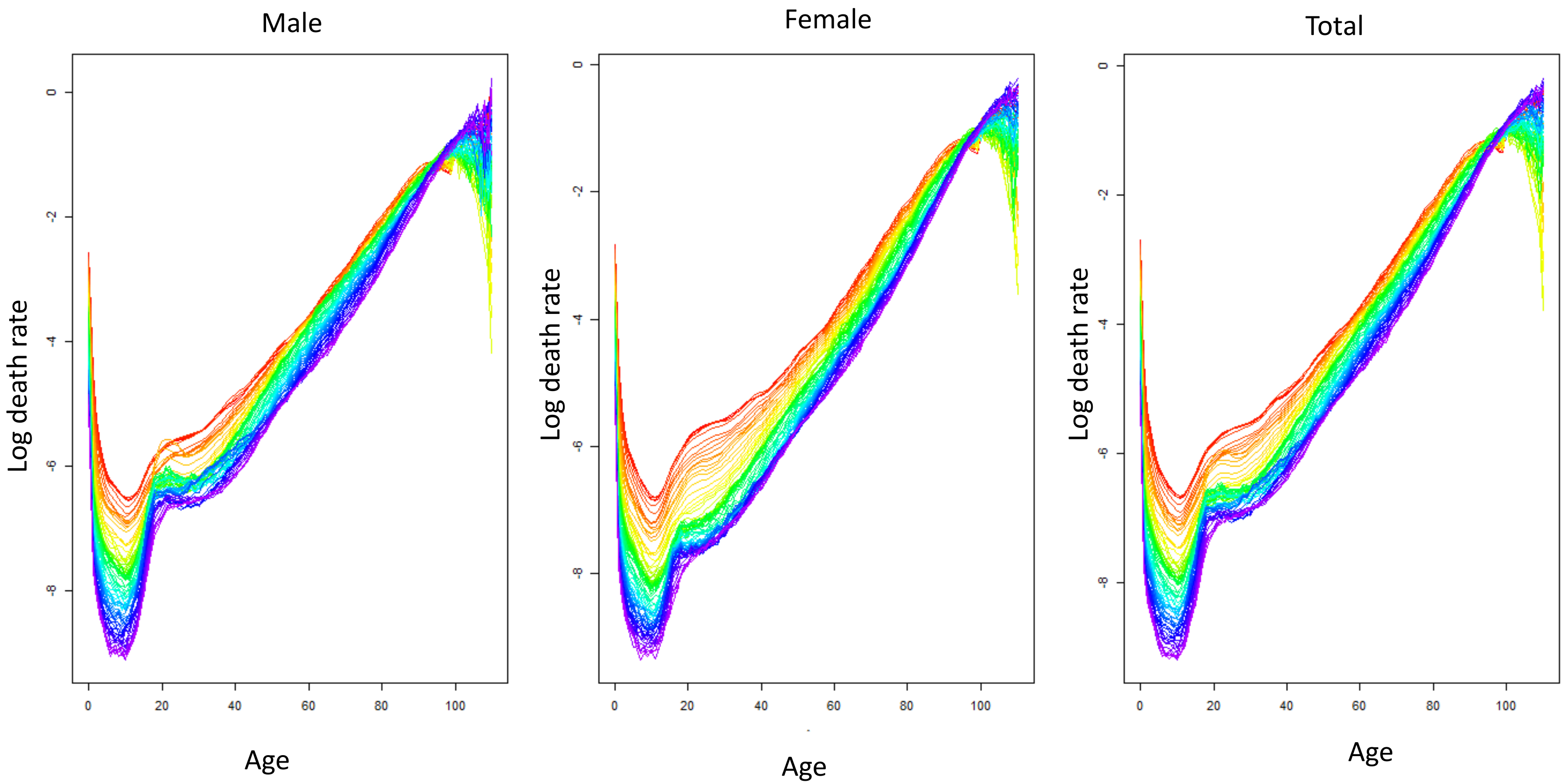
Diversified portfolio



Results

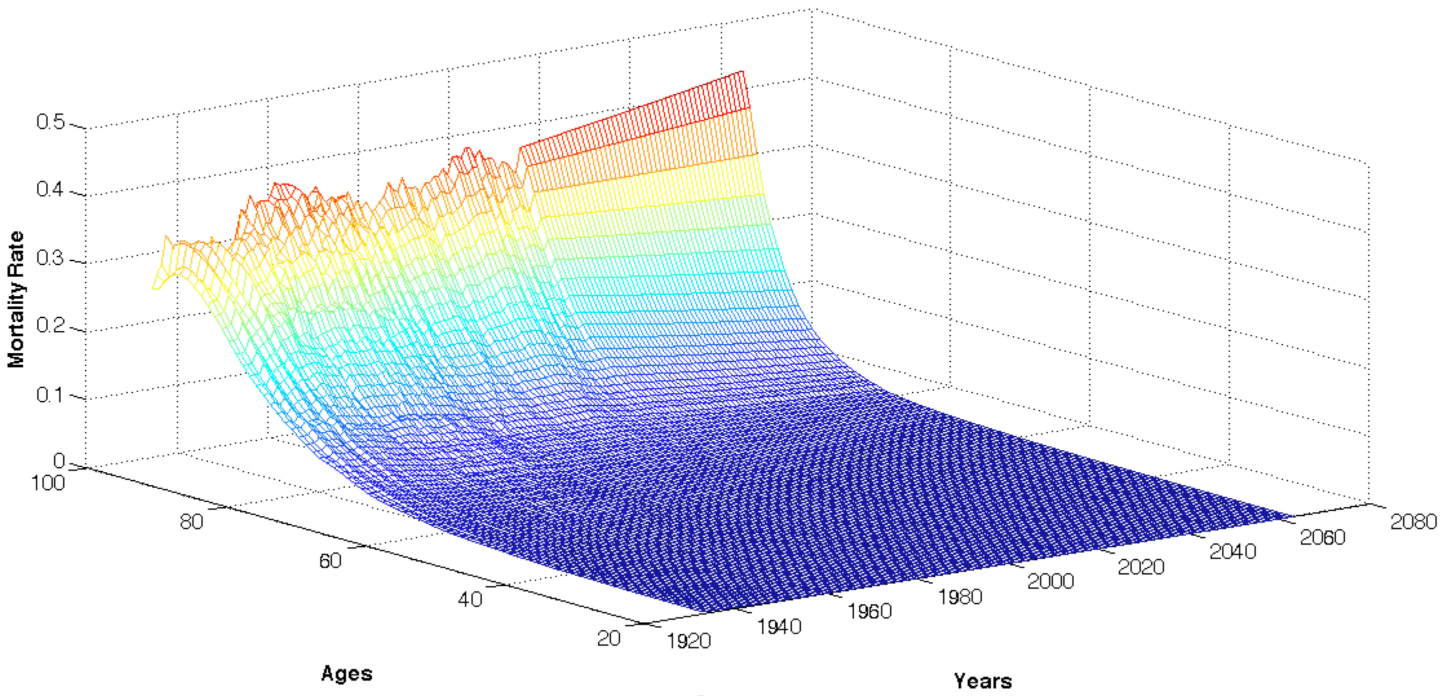


Results



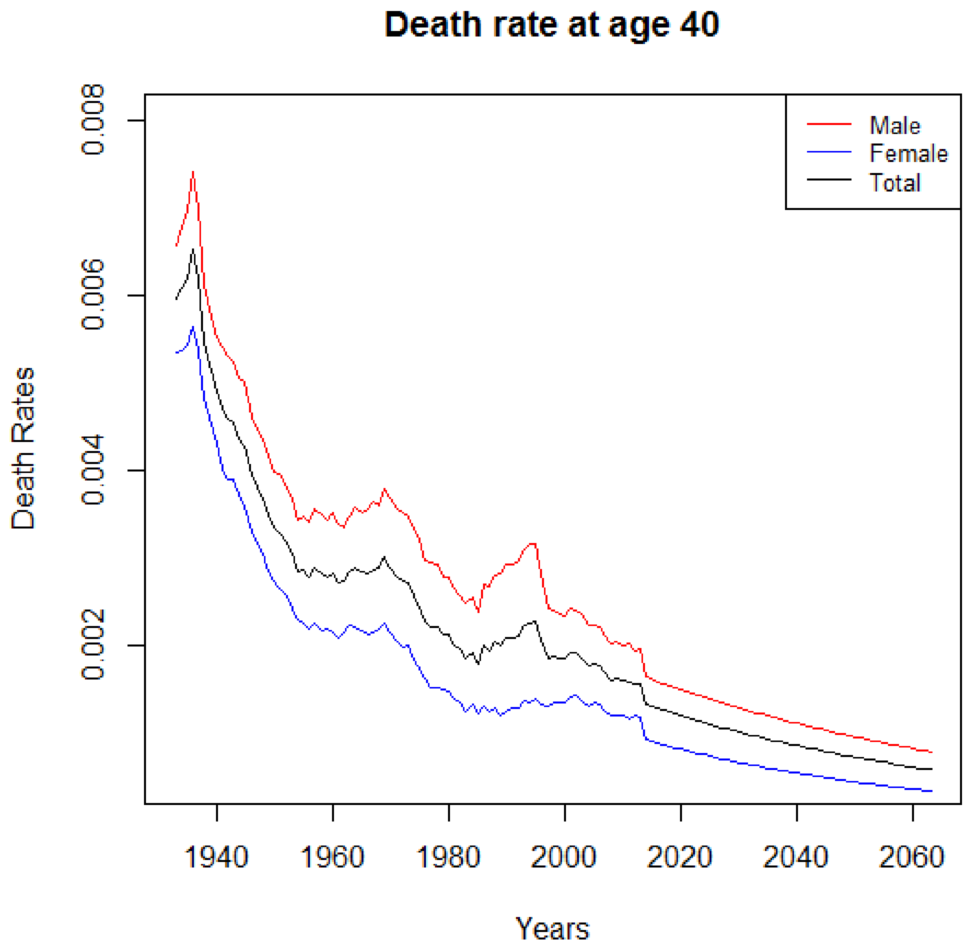
Mortality information from USA population. Available at: http://www.mortality.org/

Results

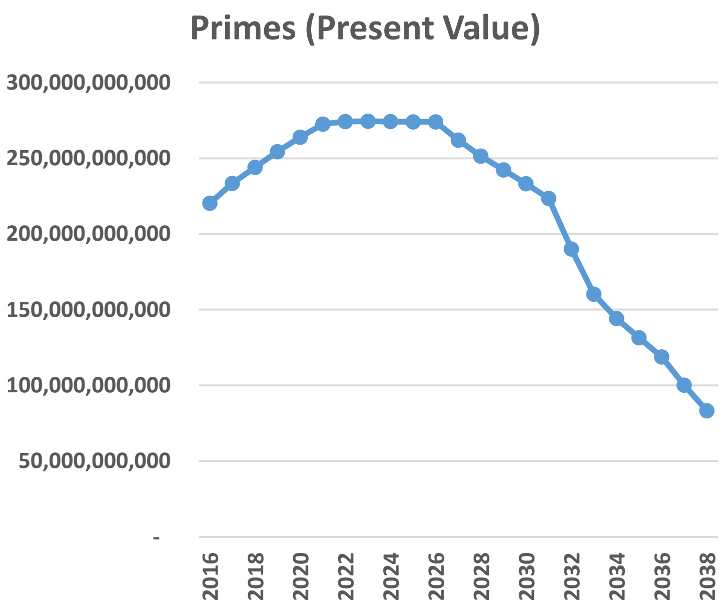


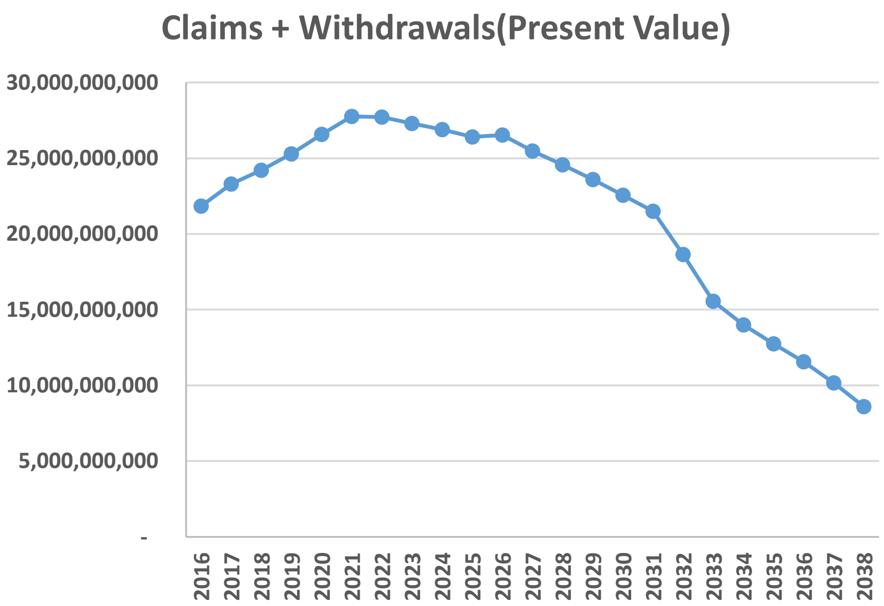
Mortality Rate
0.5
0.4
0.3
0.2
0.1
0
100
80
60
40
20
Ages
1920
1940
1960
1980
2000
2020
2040
2060
2080
Years

Results

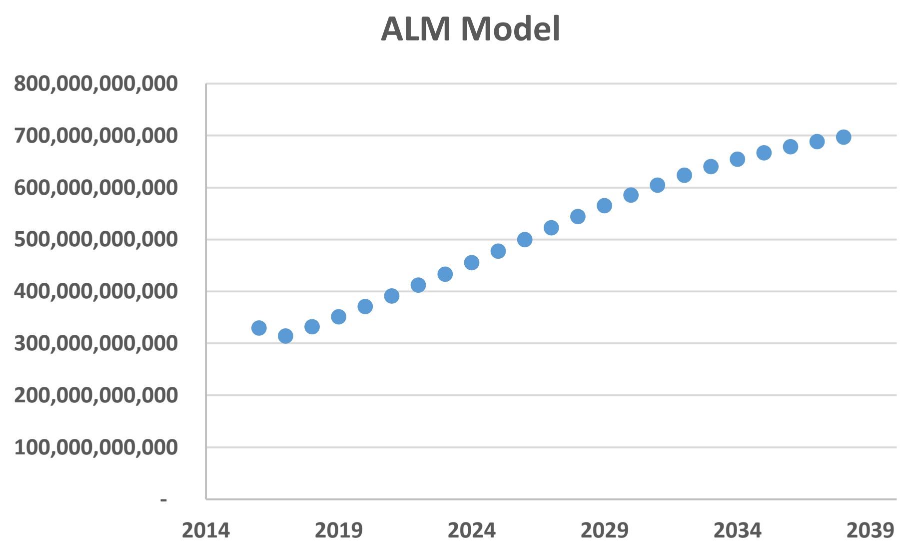

Death rate at age 40
Death Rates
0.008
0.006
0.004
0.002
1940
1960
1980
2000
2020
2040
2060
Years
Male
Female
Total

Results


Primes (Present Value)
300,000,000,000
250,000,000,000
200,000,000,000
150,000,000,000
100,000,000,000
50,000,000,000
-
2016
2018
2020
2022
2024
2026
2028
2030
2032
2034
2036
2038


Claims + Withdrawals(Present Value)
30,000,000,000
25,000,000,000
20,000,000,000
15,000,000,000
10,000,000,000
5,000,000,000
-
2016
2017
2018
2019
2020
2021
2022
2023
2024
2025
2026
2027
2028
2029
2030
2031
2032
2033
2034
2035
2036
2037
2038

Results


ALM Model
800,000,000,000
700,000,000,000
600,000,000,000
500,000,000,000
400,000,000,000
300,000,000,000
200,000,000,000
100,000,000,000
-
2014
2019
2024
2029
2034
2039

Conclusions

- It is important to know the Markowitz efficient frontier, especially in cases where the portfolio calculation is consistently. When the investment structures are created, is necessary that the CVaR is included into the Markowitz model as a severe constraint.

- Although that with the Colcap index estimation, the obtained results were favorable, it is important to note that to make predictions with this model share prices must be projected and the same with the IPC, providing an exercise which is not trivial and requires additional developments.

- Mortality estimation allows to increase the amount of investment and reinvestment through the years given that these rates tend to decrease almost in all ages, or at least in the central ages.

References

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Rumelhart, D. E., Hinton, G. E., & Williams, R. J. (1988). Learning representations by back-propagating errors. *Cognitive modeling*, *5*(3), 1.