Mathematical Modelling of a Deregulated Electricity Market With Different Production Capacities

Research Practise 2 Final Presentation

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Outline

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- $p_i \in [0, \bar{p}]$; unitary price offered by generator firm g_i, and $\bar{p}$ is a regulatory maximum price.

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with $\rho = max\{j : K_{j-1} < d\}$ and $\delta_m(i) = 1$ if $i = n_m$ and $\delta_m(i) = 0$, otherwise.

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We want to work in a Nash equilibrium situation, thus, adapting the definition of Nash equilibrium in finite mixed strategies given by [Navarro et al., 2003], we have that $F_1, F_2, ..., F_N$ is a Nash equilibrium if $\forall n \in \{1, ..., N\}$

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$$E_{F_1, F_2, ..., F_n, ..., F_N}(u_n) \geq E_{F_1, F_2, ..., \tilde{F_n}, ..., F_N}(u_n)$$

where $\tilde{F_n}$ is any other possible cumulative distribution function for the price offered by g_n.

Problem's Definition

It is known (adapting Theorem 3.1 in [Navarro et al., 2003]) that $F_1, F_2, ..., F_n$ is a Nash equilibrium if and only if for any n, the expected profit $\Phi_n(p)$ of firm g_n given that it plays the pure strategy $p_n = p$ and the other $g_i's$ play $F_i's$, is independent of p.

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Thus, we have to find an explicit formula for $\Phi_n(p)$, which is necessary if we want to obtain the equilibrium strategies $F_1, F_2, ..., F_n$, solving the differential equations system $\frac{d}{dp}\Phi_n(p) = 0$, and once the $F's$ are obtained, is possible to analyze how each type of firm plays.

Problem's Definition

An explicit formula for $\Phi_n(p)$ in the case that all firms are equal, is given in Appendix A in [von der Fehr and Harbord, 1993], as follows:

$$\Phi_n(p) = \sum_{i=1}^{N} \pi_i \{Pr[p_{n_{i-1}} \leq p \leq p_{n_{i+1}} | p_n = p]p + \int_p^{\bar{p}} \rho dF_{n_i}(\rho)\}$$

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Specific

- Understand each component of the explicit formula of $\Phi_n(p)$ in the case where all firms have the same properties.
- Define the pure strategy space and mixed strategy space for each one of the players.
- Propose a structure of the formula for $\Phi_n(p)$, when $N = 4$ using ideas of the previous case.

Outline

Results

Strategy Spaces

We noted that the pure strategy space for the player g_i is $S_i = [0, \bar{p}]$, and the mixed strategy space is

$$\Sigma_i = \{F : [0, \bar{p}] \to [0, 1] \mid F \text{ is a cumulative distribution function}\}$$

Now, if we consider that there could be ties between prices offered, the coordinator becomes another player, and its pure strategy space is given by

$$S_c = \{R : [0, \bar{p}]^N \to \{1, 2, ..., N\}^2 \mid R(p_1, ..., p_N) \text{ is a ranking of lowest prices}\}$$

Results

- We could obtain $\Phi_n(p)$, when $N = 2$.
- Then, we focused on the problem of obtaining the differential equations system $\frac{d}{dp}\Phi_n(p) = 0$, where

$$\frac{d}{dp}\Phi_j(p) =$$

$$\sum_{A\cup\{j\}\cup B=\{1,\dots,N\}} \left(\sum_{i=1}^{k_j} i\pi_{K_A+i}\right) \frac{d}{dp}\left\{(p-c_j)\prod_{a\in A} F_a(p)\prod_{b\in B}\left[\sum_{b\in B}\frac{1-F_b(\overline{p}-)}{F_b(\overline{p}-)-F_b(p)}+1\right](F_b(\overline{p}-)-F_b(p))\right\}-$$

$$\sum_{A\cup\{c\}\cup B=\{1,\dots,N\},j\in A}\left\{k_j(p-c_j)F_c'(p)\prod_{a\in A,a\neq j} F_a(p)\prod_{b\in B}(F_b(\overline{p}-)-F_b(p))\left[\sum_{b\in B}\frac{1-F_b(\overline{p}-)}{F_b(\overline{p}-)-F_b(p)}+1\right]\right\}*$$

$$\left(\sum_{i=1}^{k_c}\pi_{K_A+i}\right);\quad \forall j\in\{1,\dots,N\}.$$

- Once we got it, we solved and implemented it on *Mathematica 10.4*.

Here, we show the behavior of the F_1, F_2, F_3, in the case of $N = 3$, $(c_1, c_2, c_3) = (0, 0, 0)$, $(k_1, k_2, k_3) = (1, 2, 3)$, $\pi = (0.1, 0.2, 0.1, 0.1, 0.3, 0.2)$ and considering that the firm F_3 is which jump at $\bar{p}$, with different values of $F_3(\overline{p}-) := \lim_{p \to \overline{p}^-} F_3(p)$

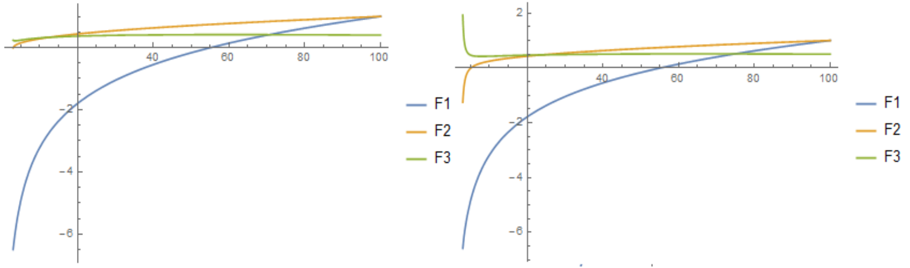

Figure: Left:$F_3(\overline{p}-) = 0.4$; Right:$F_3(\overline{p}-) = 0.5$

Results

Computational implementation

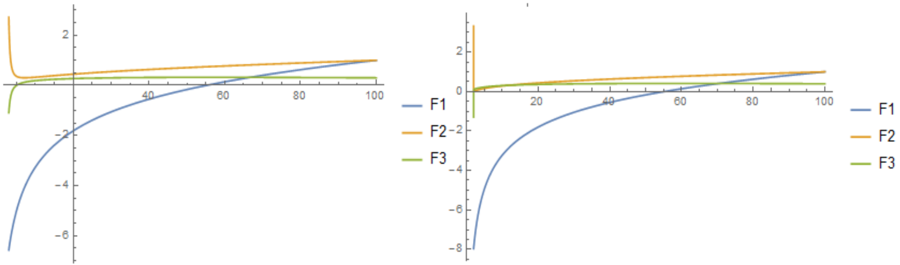

Figure: Left: $F_3(\overline{p}-) = 0.3$; Right: $F_3(\overline{p}-) = 0.395135$

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Further Work

Given that the system obtained depends of each $F_i(\overline{p}-)$, we need to know how to indentify the firm wich has the possible jump at $\overline{p}$, and the value of this jump.

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Also, we want to consider the case when the firms are divided by types, depending of their costs and capacities.

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