A Methodology for Commodity Trading in Colombia

J. Mauricio Cuscagua-Lopez Andrés F. Rojas-Amar
Ulises Cárcamo-Cárcamo

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UNIVERSIDAD EAFIT

Some Definitons

- **Commodity**: Basic good used in commerce that is interchangeable with other commodities of the same type. Most often used as inputs in the production of other goods or services.
- **Trading**: Concept that involves multiple parties participating in the voluntary negotiation and then the exchange of one's goods and services for desired goods and services that someone else possesses.
- **Derivative**: Security with a price that is dependent upon or derived from one or more underlying assets.

Futures Contracts

- **Future Contract**: contractual agreement to buy or sell a particular commodity at a pre-determined price in the future. Futures contracts detail the quality and quantity of the underlying asset and their principal characteristic is that they are standardized.

Derivatives Valuation

Let f be the price of a derivative. To calculate its price the following Partial Differential Equation must be solved [Black and Scholes, 1973]:

$$\frac{\partial f}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 f}{\partial S^2} + rS\frac{\partial f}{\partial S} = rf \qquad (1)$$

where f is the price of the Option, S is the price of the underlying, σ is the volatility of the underlying and r is the free interest rate risk.

Note

The boundary conditions depends on the Option's dynamic.

Assumptions

- The price of the underlying is a Geometric Brownian movement
- No transaction costs.
- The assets are perfectly divisible.
- The underlying pays no dividends during the life of the option.
- No arbitrage opportunities.
- The negotiation of assets is continuing.
- Free interest rate risk r is constant for all maturities.

Black-76 Model

In 1976, Fisher Black [Black, 1976] presented for the first time a variant of the Black-Scholes model which had, as its principal application, a focus for pricing options on future contracts. This model, which will be introduce further, has more light assumptions so it may be applyed to both future and forward contracts in a more suitable way.

Black-76: Futures I

The T-futures price f_t, T for a given commodity can be explained by Equation 2.

$$df_t, T = \sigma f_t, T dW_t^d \tag{2}$$

where f_t, T is the price of the T-future contract, σ the volatility of the underlying and W_t^d is a Wiener process.
The assumptions of the Black-76 model for future contracts valuation are:

- The T-futures price is perceived as a driftless lognormal process with respect to the domestic risk neutral measure.
- T will be fixed a priori.
- $f_t = f_t, T$.

Black-76: Futures II

By applying the Ito's lemma we obtain:

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 f^2 \frac{\partial^2 V}{\partial f^2} - r^d V = 0. \tag{3}$$

where, σ corresponds to the volatility of the underlying and r^d is the domestical riskless bond. V is the prife of the future and f is the T-future price.

Black-76 Model: Numerical Scheme

Following [Hull, 2006], the Finite Difference method makes a discretization of the partial derivatives in Equation (3).

$$\frac{\partial V}{\partial t} \approx \frac{V_{i+1,j} - V_{i,j}}{\Delta t}$$

$$\frac{\partial^2 V}{\partial f^2} \approx \frac{V_{i+1,j} + V_{i-1,j} - 2V_{i,j}}{\Delta f^2}.$$

Black-76 Model: Numerical Scheme

Using these substitutions, it is given in return the numerical scheme to be implemented:

$$V_{i,j} = a_j V_{i+i,j+1} + b_j V_{i+1,j} + c_j V_{i+1,j-1} \tag{4}$$

where,

$$a_j = \frac{\sigma^2 j^2 \Delta t}{2(1 + r^d \Delta t)}$$

$$b_j = \frac{1 - \sigma^2 j^2 \Delta t}{1 + r^d \Delta t}$$

$$c_j = a_j$$

Black-76 Model: Boundary Conditions

To implement the numerical scheme, boundary conditions must be defined as follow:

- For $t = \tau$, then
  $$V = \begin{cases} f - K, & \text{if } K < f \\ K, & \text{otherwise} \end{cases}$$
- For $f = f_{max}$, then $V = S_{max} - K$
- For $f = 0$, then $V = K$

where $T = \tau - t$ is the maturity time and K is the exercise price of the future.

Black-76 Model: Stability of the Numerical Scheme

In order of making the method to be stable, it is necesary that $a_j, b_j, c_j \geq 0, \forall j$.
The positivity of b_j is that $1 \geq \sigma^2 j^2 \Delta t$. The worst case is when j is takes the greatest value, i.e.:

$$1 \geq \sigma^2 \left(\frac{f_{max}}{\Delta f} \right)^2 \Delta t \leftrightarrow \frac{1}{\sigma^2} \left(\frac{\Delta f}{f_{max}} \right)^2 \geq \Delta t \tag{5}$$

Following the results obtained by Marin and Bastidas in [Marín and Bastidas, 2012], and the Von Neumann criteria, the criteria is acomplished when the amplification factor $|\epsilon| = |a_j + b_j + c_j|$ is equal to 1 when $\Delta f, \Delta t$ tend to 0.
Therefore, the numerical scheme is conditionally stable.

Black-76 Model: Aplication to Yellow Corn and Soy I

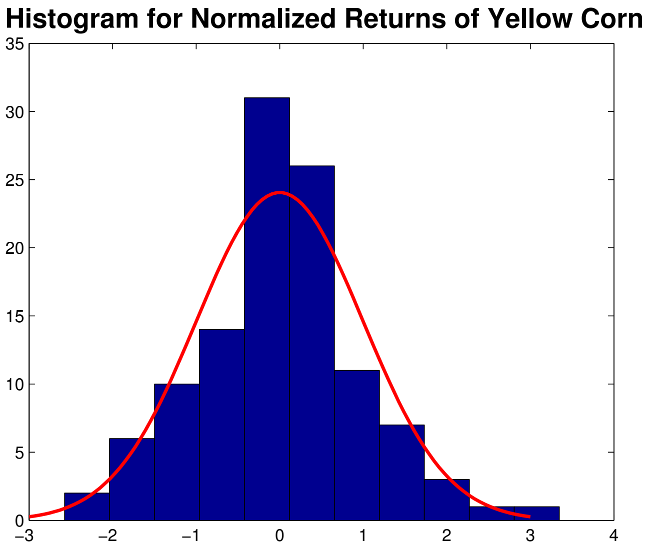

Figure 1: Adjustment of the standard normal distribution to normalized returns of yellow corn.

Black-76 Model: Aplication to Yellow Corn and Soy I

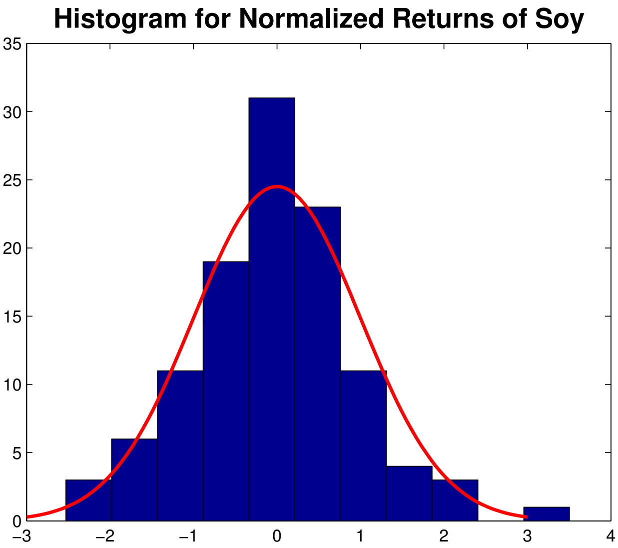

Figure 2: Adjustment of the standard normal distribution to normalized returns of soy.

Black-76 Model: Aplication to Yellow Corn and Soy I

By the implementation of Matlab, a Kolmogorov-Smirnov test was implemented. Both, the returns of yellow corn and soy, did not reject the null-hypothesis that sugests the standardized continuos returns follow a standard normal distribution. In this sense, the spot price of both yellow corn and soy can be perceived as a driftless log-normal process.

Calculations of Volatility and IBR rate

The volatility σ can be estimated by the calculation of the variance of the continuous returns for the spot price of both yellow corn and soy.
The domestic riskless bond r^d can be easely seen as the IBR rate[1] used by the bancs for reflexing the liquity of the colombian money market.

[1] Banking Benchmark. For more information about the IBR rate see [Asobancaria, 2013].

Set of Parameters

Table 1: Parameters configuration

Yellow Corn		Soy	
σ	0.96	σ	0.52
r^d	0.067	r^d	0.067
Exercise price	600	Exercise price	1100
Initial spot price	661.22	Initial spot price	1136
Maturity Time	1 year	Maturity Time	1 year

Future Valuation I

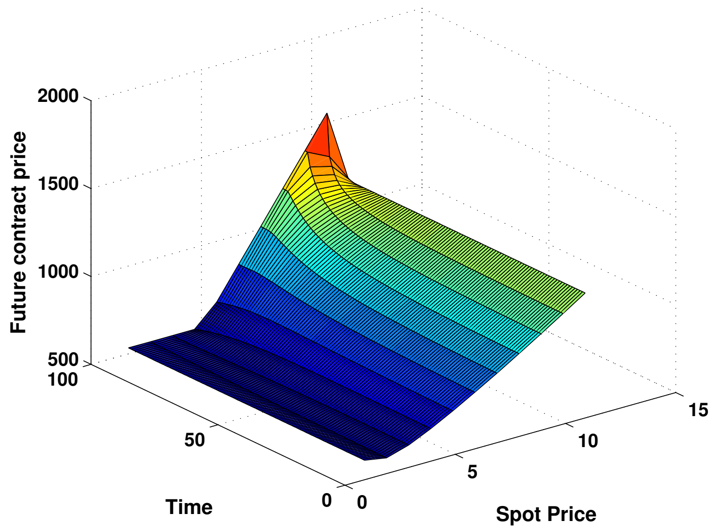

Figure 3: Mail for the valuation of a future contract over a unit of yellow corn for one year as maturity time. The calculated price for the future contract is \$717.12 COP.

Future Valuation II

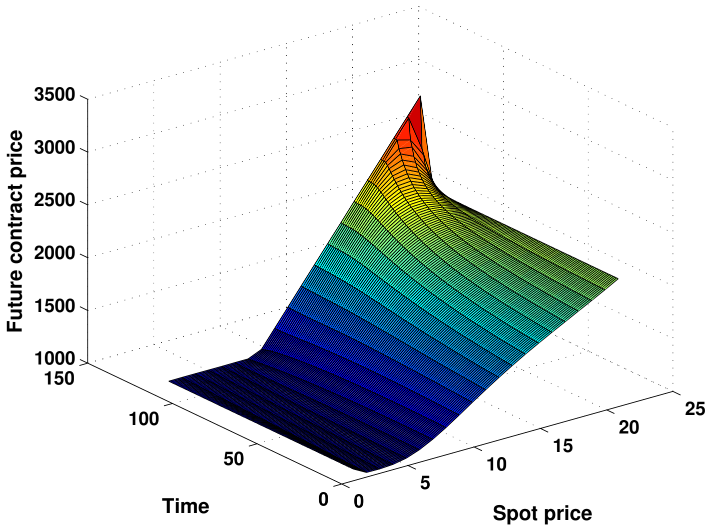

Figure 4: Mail for the valuation of a future contract over a unit of soy for one year as maturity time. The calculated price for the future contract is $\$1,238.3\ COP$.

Method Convergence I

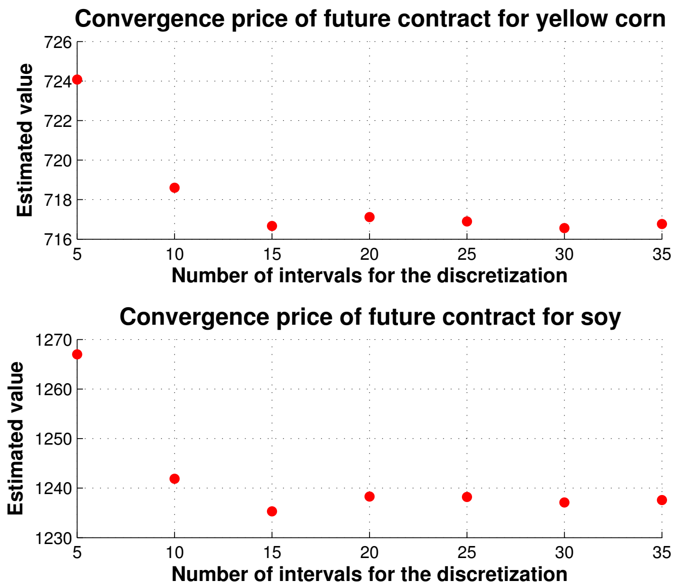

Figure 5: Convergence of the price estimated as the number of nodes in the mail increases.

Conclusions and Further Work I

- The Black-76 model can be applied to soy and yellow corn trading where the market uses the IBR rate instead of a domestic riskless bond.
- It was shown the good behavior of the Black-76 model applied to historical data that follows the assumptions established by the model.
- To verify the monotonicity, positivity, consistency, stability and convergence of the numerical scheme.

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