EAFIT University

Research practice 1

CM0436

Vessel Extraction Using the Buckmaster-Airy Filter

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1 Problem Description

Vessel segmentation and extraction are important tools in the analysis of biomedical images of the circulatory blood vessels which are key components to automated radiological diagnostic system and in general for diagnosis of vascular disease, such as malformations. Partial differential equations have several applications, especially in models for wave motion. Using two partial differential equations a filter can be created considering the linear dispersion of the tones of black and white images. The linear dispersion can be given in terms of the two-dimensional Airy equation

$$\frac{\partial}{\partial t}P(x,y,t) = \eta_1\left(\frac{\partial^3}{\partial x^3}P(x,y,t)\right) + \eta_2\left(\frac{\partial^3}{\partial y^3}P(x,y,t)\right) \tag{1}$$

with the initial condition

$$P(x,y,0) = \delta\left(x - X\right) + \delta\left(y - Y\right) \tag{2}$$

where δ represents the Dirac delta function, x and y are the coordinates of a pixel of the image. For a more efficient algorithm the image is divided in a desired number of submatrix, each submatrix is called a window, X and Y are the coordinates of the center of the window where (x, y) is located.

By applying the Fourier transform to Equation 1 with the initial condition in Equation 2 and assuming $\eta = \eta_1 = \eta_2$, we can obtain the explicit solution

$$P(x,y,t) = \frac{1}{3}\left(\frac{3^{1/3}Ai\left(\frac{3^{2/3}(-x+X)}{3\eta^{1/3}t^{1/3}}\right)Ai\left(\frac{3^{2/3}(-y+Y)}{3\eta^{1/3}t^{1/3}}\right)}{\eta^{2/3}t^{2/3}}\right) \tag{3}$$

Performing the change of variable $\eta = \frac{\sigma^3}{t}$, Equation 3 takes the form of

$$P(x,y,t) = \frac{1}{3}\left(\frac{3^{1/3}Ai\left(\frac{3^{2/3}(-x+X)}{3\sigma}\right)Ai\left(\frac{3^{2/3}(-y+Y)}{3\sigma}\right)}{\sigma^2}\right) \tag{4}$$

We will also use the Buckmaster equation [1] in two dimensions which describes thin viscous fluid sheet flow and it is given by Equation 5, but it is non-linear so it is not possible to find an analytical solution.

$$\begin{aligned}\frac{\partial}{\partial t}u(x,y,t) = &\left(\frac{\partial^2}{\partial x^2}u(x,y,t)^4\right) + \left(\frac{\partial}{\partial x}u(x,y,t)^3\right) \\ &+ \left(\frac{\partial^2}{\partial y^2}u(x,y,t)^4\right) + \left(\frac{\partial}{\partial y}u(x,y,t)^3\right)\end{aligned} \tag{5}$$

We will construct the filter named here the Buckmaster-Airy filter applying the spatial operator Buckmaster (Equation 5) to Equation 4. This filter will be applied to a given biomedical digital image via convolution. For practical applications the filter must be discretized.

2 Objectives

2.1 General Objectives

Design a technique for vessel extraction from biomedical images using the Buckmaster-Airy filter.

2.2 Specific Objectives

- Develop a filter for biomedical images processing using partial differential equations.
- Compare the performance of the Buckmaster-Airy filter with other filters obtained from partial differential equations.
- Present the results of the project in an international conference.

3 Preceding researches

Nowadays there are many different techniques for vessel extraction, one of the oldest references for works about image analysis and computer vision problems is provided by Binford [2], who introduced the Generalized Cylinders Model in vision application. Other models can be categorized in six different types [3]:

1. Pattern recognition techniques
2. Model-based approaches
3. Tracking-based approaches
4. Artificial intelligence-based approaches
5. Neural network-based approaches
6. Miscellaneous tube-like object detection approaches

Even if the main target is the extraction of blood vessels, many of these models can be applied to obtain a segmentation for tubular objects with similar characteristics to vessels. The best method for the extraction may vary in terms of the image quality and general image artifacts such as noise. Some methods need pre-processing prior segmentation while others apply post-processing to overcome the problems arising from over segmentation.

4 Justification

Vessel segmentation is not an easy task because it depends of vessels width, image resolution, contrast, brightness and other image features. Currently there are different types of methods to perform the vessel segmentation but there is not a method that works with every medical image modality. Using partial differential equations and special functions of mathematical physics we will create a filter which can be used to improve visualization of blood vessels for black and white images. This project introduces a filter that detects filaments of biomedical images, which may be efficient and versatile.

5 Scope

The scope of this project is to develop a technique for circulatory blood vessel extraction, which in the best of the cases will be new. According to the method that is going to be developed this filter will focus only in vessel segmentation in black and white images and will not need pre-processing or post-processing of the image. Also it is important to compare the performance of the Buckmaster-Airy filter with other filters obtained from partial differential equations.

6 Methodology

1. Computational propaedeutics with Maple: This stage came before the Course CM0436, Maple software was required, the tasks were provided by the project tutor.

2. Reading relevant papers: This activity happens along the project to achieve the required knowledge for the research, databases (provided by EAFIT University) will be required.

3. Construction of the model: In this part the student will apply the knowledge of partial differential equations.

4. Computational implementation of the model: The computational implementation will be done using the ImageTools package included in Maple 11, also to prove the efficiency, the filter will be applied to different images and its performance will be compared to other filters.

Table 1: Schedule

Activity	Description	Start	End
Literature review	Read papers with related topics and make a literature review for contextualization in the subject	Week 1	Week 16
Computational implementation	Perform the computational implementation of different types of partial differential equations	Week 1	Week 4
Proposal	Write the project proposal and make an oral presentation	Week 3	Week 4
Oral report	This report will take place every two weeks to keep the project tutor updated with the work of the student in the research	Week 3	Week 13
Progress	Make an oral progress report of the research	Week 10	Week 10
Write paper	Writing a paper showing the results of the research and apply to present the results in an international conference	Week 4	Week 11
Project report	Make a final report of the research presenting information included in the paper	Week 16	Week 16
Project presentation	Make oral presentation of the results of the accomplished work during the semester	Week 19	Week 19

5. Writing the paper: This is the final phase of the project, the student will write the paper based in the information about the topic and the results of the computational implemented model.

7 Schedule

See Table 1.

8 Resources

The project will not generate any cost so it is not necessary to quantify a required budget, but it is important to notice that the access to databases and the Maple software license are provided by EAFIT University, but this resources are open for students and researchers linked to this institution.

9 Intellectual Property

Given the internal regulations on intellectual property of EAFIT university[4] it is important to clarify that the product of this research is going to be developed between the student Valentina Sánchez Bermúdez and the project tutor Juan Fernando Ospina Giraldo, however all the author's royalties of the paper belong to the student.

References

[1] E. A. Hussain and Z. M. Alwan, "The Finite Volume Method for solving Buckmaster's Equation, Fisher's Equation and Sine Gordon's equation for PDE's," in *International Mathematical Forum*, vol. 8, pp. 599–617, 2013.

[2] T. O. Binford, "Visual perception by computer," in *IEEE conference on Systems and Control*, vol. 261, p. 262, 1971.

[3] C. Kirbas and F. Quek, "A review of vessel extraction techniques and algorithms," *ACM Computing Surveys (CSUR)*, vol. 36, no. 2, pp. 81–121, 2004.

[4] Universidad EAFIT, "Reglamento de Propiedad Intelectual". 2009.