# Real Options Valuation for Mining Projects Using a Proposed Numerical Schema Based on Finite Difference Method*

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Research practice I: Project proposal

## I. Problem Formulation

The real options valuation has been an actual topic of interest for economy and finances. A real option itself, is the right (but not the obligation) to undertake certain business initiatives [2], such as deferring, abandoning, expanding, staging, or contracting a capital investment project. The literature about Real Options valuation is relatively recent. The foundation comes from financial option pricing theory beginning with Black and Scholes (1973) [3], Merton(1973) [4], Cox and Ross(1976), Ross and Rubinstein(1979)[5].

As we said, in 1973, F. Black, M. Scholes y R. C. Merton developed a theory model which describes conditions that must be accomplished the price of all option and let us calculate the respective value. Black, Scholes and Merton assumed that the price of the underlying is a geometric Brownian movement. Then, the price $S_t$ satisfies the SDE[1] bellow:

$$dS_t = \mu S_t dt + \sigma S_t dB_t \tag{1}$$

where $B_t$ is a SBM[2].

In addition, the model has the following assumptions:

- No transaction costs.
- The assets are perfectly divisible.
- The underlying pays no dividends during the life of the option.
- No arbitrage opportunities.
- The negotiation of assets is continuing.
- Free interest rate risk $r$ is constant for all maturities.

Using this assumptions and the Ito's lemma it's shown that the price of an option $f(t, S)$ must satisfies the following PDE[3]:

$$\frac{\partial f}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 f}{\partial S^2} + rS\frac{\partial f}{\partial S} = rf \tag{2}$$

where $f$ is the price of the option, $S$ is the price of the underlying, $\sigma$ is the volatility of the underlying and $r$ is the free interest rate risk.

The boundary conditions depends on the kind of option.

Applications for Equation (2) are quite diverse today, specially for all kind of options valuation. For our interest, we will study the mining area and how the real options valuation for mining projects are described by variations of the Black-Scholes equation.

In the case of real options valuation, the opportunity to invest in the expansion of a firm's factory or alternatively to sell the factory, could be an example of the importance to calculate the real option value which is really complex knowing that we don't have sufficient information about the factory that describes what can happen in the future. The other problem is that while more complex is the PDE is harder to implement a numerical scheme to solve it and that's why this is an area of interest for many people today. Following [6], the Black-Scholes Equation (2) can be expressed as the Heat Equation but taking out the source term. It can be also solved using numerical methods but the case for mining projects that describes the real option price is more complex.

In [5], Colwell, Henker and Ho explains that there are different ways to see the valuation of mine, i.e., the value of the mine depends on the state of the mine: Opened, Closed

*The structure and the template was based on Velazquez(2015) [1]
[1]Stochastic Differential Equation
[2]Dimensional Standard Brownian Movement
[3]Partial Differential Equation

or Operated. The corresponding options can be intuitively describes like:

- **Opened Mine Option:** If the price of gold, $S$, is high enough relative to the average extraction cost, $A$, then the mine operator will exercise the option to open and operate the mine.
- **Closed Mine Option:** If the price of gold, $S$, is lower than the average extraction cost, $A$, then the mine operator will exercise the option to mine but not necessary abandoned.
- **Abandon the Mine Option:** All maintenance costs cease and the mine becomes worthless.

They also compares the ownership of a mine with the option to open, close and abandon a mine is analogous to an American call option on the gold in the mine. The price of gold, $S$, is analogous to the price of the underlying asset, and the average extraction cost, denoted by $A$, is analogous to the option's strike price. Similar to financial option pricing, the other relevant parameters are the volatility of the price of gold, $\sigma$, the free interest rate risk, $r$, and the convenience yield for gold, $c$. In this model the only stochastic variable is the price of gold.

As we said, every case has different PDE for the real option valuation. Equations in [5] describe the price of the mine depending on the state of the same. What we propose to do is to use a transformation to simplify the number of terms from the original PDE and then, when it's easier to process, solve the alternative PDE and get the price or the real option.

## II. Objectives

### A. General Objective

To implement a transformation that simplifies a Partial Differential Equations which describes the value of real options for mining projects and check its efficiency in calculating real options values.

### B. Specific Objectives

- Understanding the Options' dynamic as well as their computational implementations.
- Propose and implement a transformation for the mentioned PDE to reduce the number of terms that it has.
- Implement the proposed numerical schema for real options valuation.
- Evaluate the convergence, consistency and stability of the numerical schema.

## III. Preceding Research

In 1972, Cox, Ross, and Rubinstein proposed a methodology exposed in [7], chapters 12 and 20. The binomial tree valuation approach involves dividing the life of the option into a large number of small time intervals of length $\delta t$. It assumes that in each time interval the price of the underlying asset moves from its initial value of $S$ to one of two new values, $S_u$ and $S_d$. The method works with the probabilities to move "*up*" denoted $p$, or "*down*" denoted $1-p$.

In 1977, Phelim Boyle applied the Monte Carlo simulation model for option pricing (for European options) [8]. In 1997, M. Broadie and P. Glasserman implemented the Monte Carlo simulation for american option pricing [9]. Finally, In 2001 F. A. Longstaff and E. S. Schwartz developed a practical Monte Carlo method for pricing American options[10].

The method can be described in two phases:

- **First:** A backward induction process is performed in which a value is recursively assigned to every state at every time step. The value is defined as the least squares regression against market price of the option value at that state and time. Option value for this regression is defined as the value of exercise possibilities (dependent on market price) plus the value of the time step value which that exercise would result in.
- **Second:** When all states are valued for every time step, the value of the option is calculated by moving through the time steps and states by making an optimal decision on option exercise at every step on the hand of a price path and the value of the state that would result in. This second step can be done with multiple price paths to add a stochastic effect to the procedure.

Perhaps one of the most useful numerical methods ever used has been the Finite Difference method. As we found in [7], this is a numerical method used in mathematical finance for option valuation. In 1977, it was first applied for option pricing by Eduardo Schwartz [11]. Finite difference methods has been used to price options by approximating the PDE that describes how an option price evolves over time by a set of difference equations. Once the equation is taken in differences, it can be solved iteratively to calculate the price for the option. The dynamic of the numerical schema can be describe as following:

1) Maturity (when the option's time has lapsed) values are simply the difference between the exercise price of the option and the value of the underlying at each point.
2) Values at the boundary prices are set based on moneyness or arbitrage bounds on option prices.
3) Values at other lattice points are calculated recursively, starting at the time step preceding maturity and ending at time 0 (The iterative technique depends on whether the explicit or implicit method is implemented).
4) The value of the option today, where the underlying is at its spot price, or at any time/price combination, is then found by interpolation.

Finite Difference method will be the technique to be considered as the base for a our proposed numerical scheme.

## IV. Justification

Generally, options valuation has been an issue in recent days because of its use for reducing investment risk, for that reason the research of better numerical methods for calculating option prices (particularly real options) has increased in this our time.

Focusing on Finite Difference method, there are many characteristics that make it subject of study (execution time, stability, convergence, etc). From the literature [7] there are conditions that must be accomplished to guarantee the functionality of the method.

Other important fact is that as the price step is closer to zero, the computation time considerably increases.

The areas of knowledge necessary for the solution of the problem are:

- Stochastic Processes
- Numerical Methods
- Multivariate Calculus
- Financial Mathematics
- Statistic

The project seeks to propose an alternative formulation for PDE's which describes the value for real options and improve their estimation time with a numerical schema resulting from a variable change that is shown in Section VI.

The results of the project can help in financial and economic fields for real options valuation. It also provides the possibility to research the use of the numerical schema for any kind of financial asset valuation which are described by a PDE.

## V. Project Scope

The principal approaches of the project are:

- An alternative expression for Equation (2) and get its numerical solution.
- Implementation of different numerical schema for pricing real options.
- Determine the convergence, stability and consistency criteria for the proposed numerical schema.
- Apply the numerical schema to real options valuation for mining projects.
- Generalization of the numerical schema for application to any PDE which describes the value for financial instruments (particularly the real options).

The project will focus on application of the alternative expression for Black-Scholes Equation and the implementation of the same to Real Options valuation for mining projects. The issue of greatest interest in the project is the evaluation of all conditions of convergence, stability and consistency for the numerical schema.

The generalization of the proposed numerical schema will be left as future work or extra work option in the project (given that can be treated during the time budgeted for extra work project). The reason of this decision is the effort that will be required to generalize the process for any kind of financial issue independent of its dynamic.

## VI. Methodology

The first step is to define the equation to be considered as the transformation of the PDE. Let us take the following change of variable

$$\begin{aligned} H &= \mathrm{e}^{r(\tau-t)} f \\ X &= \mathrm{e}^{r(\tau-t)} S \end{aligned} \tag{3}$$

Once the expressions in Equation (3) had been treated we will obtain the alternative PDE to find $H$. Note that $H$ is a function of $X$ and $t$. When we have the alternative PDE, it will have less terms than the original PDE, so we can apply the dynamic of Finite Difference method to the new PDE, and with the value of $H$, we can find $f$ from the first expression in Equation (3).

The numerical schema will be used then in PDE's which describe the value of a mining project. As examples, numerical calculation will be taken from [12] and [5].

For simulation, the software MATLAB®2015a version will be used.

## VII. Activity Schedule

An estimated schedule of the different phases of the project is proposed in Table I. According to the currently deadlines of the Research Practice I course of Mathematical Engineering, the report's deadlines are shown on Table II.

| Activity | Estimated Time Range |
|---|---|
| Literature review | Jul. 21 - Aug. 8 |
| Applying the numerical scheme to Black-Scholes' equation. Make simulations and tests. | Aug. 10 - Aug. 21 |
| Determine the convergence, consistency and stability of the numerical schema | Aug. 21 - Oct. 3 |
| Applying the numerical scheme to specifics PDE from the literature of interest. Make simulations and tests. | Oct. 3 - Oct. 15 |
| Extra work. | Oct. 16 - Nov. 3 |

Table I

| Activity | Deadline |
|---|---|
| Project Proposal Report | Aug. 7th |
| Project Proposal Presentation | Aug. 14th |
| Oral Progress Report | Sep. 25th |
| Project Report | Nov. 6th |
| Final Presentation | 19th Academic week |

Table II: These dates were taken from the course's website: http://www1.eafit.edu.co/asr/courses/research-practises-me/2015-2/index.html

## VIII. Budget

The project will not require any financing.

## IX. Intellectual Property

In accordance with EAFIT university's property ruling [13], the patrimonial rights over all academic products resulting from the actual project belong to:

- Juan Mauricio Cuscagua Lopez
- Freddy Hernán Marín Sanchez
- EAFIT University

The ruling sets that utilities obtained through commercialization of any product of the project must be assigned as the following proportions:

- Juan Mauricio Cuscagua Lopez: 25%
- Freddy Hernán Marín Sanchez: 25%
- EAFIT University: 55%

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