ANALYSIS OF PROCESSES CAPABILITY USING THE SKEWED NORMAL DISTRIBUTION

Research Practice I

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Problem Formulation:

The capability of a process refers to the range within which the natural variation occurs, that is, the variation produced by random causes only [1]. The potential process capability index (C_p) is defined as the ratio of the natural variation of a process and specifications [1, 2]:

$$C_p = \frac{USL - LSL}{6\,\sigma}$$

where USL is the upper specification limit, LSL is the lower specification limit, and σ is the standard deviation of the process.
Moreover, it is interesting to evaluate the losses caused by the breach of customer specifications, and it is used a Process Capability Index (PCI) capable of measuring actual performance or potential characteristics of the process (C_{pm}) in relation to the value nominal and pre-established specification limits [3]

$$C_{pm} = \frac{USL - LSL}{6\,\sqrt{\sigma^2 + (\mu - T)^2}}$$

where μ is the mean of the process and T is the target specification of the quality characteristic.

The estimation of these PCIs requires that the process data are specific and to follow a normal distribution. However, non-normally distributed processes are common in practice. This forced academicians and practitioners to investigate the characteristics of process capability indices with non-normal data. Literature review shows that the research for PCIs under non-normality has been grouped into two main streams [4, 5]: (1) examination of PCIs and their performances for various underlying distributions and (2) construction of new generation process capability indices and development of new approaches specially tailored for non-normally distributed outputs. Within this case we have: data transformation methods, development of quality control procedures for certain non-normal distributions, distribution fitting for empirical data, development of distribution-free procedures, and construction of new PCIs.

The most commonly used methods for estimating process capability indices on non-normal processes are described in [5, 6]:

- **Clements's Percentile Method:** It calculates the indices using a family of Pearson Curves. This method consists of estimating process capability indices as if they were normal but with two simple slight changes: Instead of using values of percentile points from normal distributions it takes them from non-normal distributions, and instead of taking the mean as the measure of central tendency it uses the median.

- **Burr's Percentile Method:** This is a change to Clements's Percentile Method by substituting the percentiles in the family of Pearson Distribution Curves with an appropriated Burr XII distribution.

When output of process is not normally distributed, then it is required the assumption of an empirical distribution of the data to analyze. Commonly used probability distributions are characterized by being positive asymmetric. The distributions used to describe the process outputs are: Generalized Pareto with 2 parameters, Weibull, Log-Normal and Beta distributions [4, 5, 6].

In the literature review was found that negative asymmetrical distributions have not been considered yet, omitting the proper study of processes that may have this behavior. Among the probability distributions with negative asymmetric behavior, the Normal Skewed distribution stands out. The probability density distribution associated to a random variable with a Skewed Normal distribution is as follows:

$$f(x) = \frac{1}{\omega\pi} e^{-\frac{(x-\xi)^2}{2\omega^2}} \int_{-\infty}^{\alpha\left(\frac{x-\xi}{\omega}\right)} e^{-\frac{t^2}{2}}\, dt$$

where $\xi \in \mathbb{R}$ is a position parameter, $\omega \in \mathbb{R}^+$ is a scaling parameter and $\alpha \in \mathbb{R}$ is a shape parameter. Note that the Skewed Normal distribution is just an extent of Normal Distribution by adding a shape parameter that gives three possible distribution shapes: positive asymmetry, negative asymmetry and symmetry itself [7].

Parameter estimation is accomplished by means of the maximum likelihood method and numerical methods [8]. The equation to be solved is the following:

$$L(\xi, \omega, \alpha) = \frac{2^n}{\omega^n} \prod_{i=1}^{n} \phi\left(\frac{x_i - \xi}{\omega}\right) \Phi\left(\alpha \frac{x_i - \xi}{\omega}\right)$$

where $\phi\,(.)$ and $\Phi\,(.)$ stand for probability density function and cumulative distribution function for standard normal respectively. Several important fields of knowledge have intensively applied this statistical distribution like Risk Assessment and Capital Assignation commonly found in Actuarial Science, Insurance and Risk Management [9]. Nevertheless, this distribution has not been used in the capability analysis of industrial processes.

Main Goal:

Develop capability indices for processes with non-normal data using Skewed Normal distribution.

Specific Goals:

- Identify methods to estimate process capability indices associated with non-normal data.
- Select one of these methods and adapt it to the Skewed Normal distribution.
- Develop the proposed methodology in a programming language.
- Compare the proposed methodology performance against conventionally used methods reported in literature to establish advantages and drawbacks.

Background:

In conducting the literature review, the advances that have been made in the research of process capability indices for non-normal data is shown in Table 1:

Table 1. Background of process capability indices for non-normal data.

Author	Topic	Contribution
Liu and Chen [10]	Process capability analysis with non-normal data using the Burr XII distribution	Development of a new estimation method called "Burr method for non-normal data," modifying the Clements method and using the Burr XII distribution.
Ahmad, Abdollahian, and Zeephongseku [11]	Comparison of proposed methods to calculate process capability indices to non-normal data	Shown, using simulation techniques, that the method of the cumulative distribution function based on the Burr distribution, produces better estimates of process capability index. Additionally, they show that the method of Burr is better in accuracy than the method Clements.
Abbasi [12]	Estimation of capability indices of non-normal processes using neural networks.	Development of a neural network with a hidden layer to estimate index capacity to non-normal processes, using as input variables the coefficients of skewness, and kurtosis, and the lower specification limit.
Hosseinifard, Abbasi, Ahmad, and Abdollahian [13]	Estimation of capability indices non-normal processes.	Approach to the use of transformation technique called "root transformation" in the estimation of process capability index when the data are not normal. The method is validated using simulated data from the Gamma, Beta and Weibull distributions.
Leiva et al. [14]	Capability indices for Birnbaum-Saunders processes.	Development of the theory related to the estimation and inference of process

		capability index, whose data come from the Birnbaum-Saunders distribution. In addition, the methodology is implemented in the software R.

Justification:

The data referring to quality control in processes do not always fit properly into positive asymmetric or just symmetric distributions. Hence, by their own nature associated to external factors and for flexibility many data also require negative asymmetric distributions. Therefore it is necessary to formulate new process capability indices, or adapt existing, with probability distributions also consider negative skewness. Being Skewed Normal distribution a good candidate to consider.

Thus, the importance and originality of this research is:

1. According to the systematic literature review, there are no proposals for estimating process capability indices under Skewed Normal Distribution.
2. The asymmetrical flexibility given by the Skewed Normal distribution will render its worth by giving robust process capability indices and making easier the data fitting.

Scope:

In this research project will be developed a methodology for building process capability indices using Skewed Normal distribution. This methodology will be implemented by means of programming language but validated with experimental data instead of using real data.

Proposed Methodology:

The main steps are as follows:

Technical literature review to identify used methods to estimate process capability indices applied to non-normal processes. It is important to establish how they are defined, tested and applied to characterize process performance.

Then, several methods will be first selected for adaptation to the Skewed Normal distribution and then implemented by means of an appropriated programming language.

Afterwards, some skewed normal data will be generated and the results obtained with the implemented method will be compared against conventionally used method results reported in literature to establish advantages and drawbacks. Some conclusions will be finally drawn.

As a special note, there will be several tutoring meetings during the academic semester to evaluate project contents, progress, teamwork, results and advances. This approach will make this project to succeed.

Activity Schedule:

The following tables sketch activities, semester weeks and dates that must be fulfilled in order to accomplish this project.

Table 2 Activity schedule during research.

Activity	Semester/Week	Dates
Literature review.	1 - 5	July 21 – August 23
Methods adaptation to the Skewed Normal Distribution.	6 - 9	August 24 – September 20
Implementation by means programming language.	8 - 11	September 7 – October 4
Selected methods performance comparison against conventionally used methods reported in literature.	12 - 14	October 5 – October 25
Project report.	15 - 16	October 26 – November 8
Project presentation.	17 - 19	November 9 – November 27

Table 3 Key dates during Research Practice I.

Activity	Week	Date
Proposal report.	3	August 7
Proposal presentation.	4	August 14
Oral progress report.	10	September 25
Project report.	16	November 6
Project presentation.	19	November 27

Data for Table 3 was taken from the following web page related to this research course: http://www1.eafit.edu.co/asr/courses/research-practises-me/2015-2/index.html

Budget:

This research does not require financing.

Intellectual Property:

Susana Agudelo Jaramillo, Myladis Rocio Cogollo Flórez and Juan Miguel Cogollo Flórez share intellectual property in this research equally.

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