IMPLEMENTATION OF FINITE ELEMENTS METHOD ON A DIFFUSION-ADVECTION PROBLEM

Research practise I proposal presentation

Obed Ríos-Ruiz
oriosru@eafit.edu.co

Advisor: Jairo Alberto Villegas-Gutierrez
javille@eafit.edu.co

EAFIT University

August 14, 2015

Outline

Outline

Introduction

General problem description

There are several physical phenomena underlying the transportation or transference of chemical particles inside a physical system. Most of the times such phenomena is due to two processes: *Diffusion* and *Advection*.

Introduction

General problem description

Advection(in atmospheric science) means a change in a property of a moving mass of air because the mass is transported by the wind to a region where the property has a different value.[1]

[1]Encyclopedia Britanica, http://www.britannica.com/science/advection, consulted on 2015-08-01.

Introduction

General problem description

Advection(in atmospheric science) means a change in a property of a moving mass of air because the mass is transported by the wind to a region where the property has a different value.[1]

Diffusion(in physics) is a process resulting from random motion of molecules by which there is a net flow of matter from a region of high concentration to a region of low concentration.[1]

[1] Encycopledia Britanica, http://www.britannica.com/science/advection, consulted on 2015-08-01.

Introduction

General mathematical problem

Consider the general nonconservative partial differential equation describing the advection-diffusion phenomena in a medium:

$$\phi \frac{\partial c}{\partial t} + \mathbf{u} \cdot \nabla c - \nabla \cdot (\mathbf{D} \nabla c) = \bar{c} q, \qquad \mathbf{x} \in \Omega, \quad t \in [0, T]$$

where c is the chemical species concentration, Ω is the physical domain, $\mathbf{u}$ is the *Darcy* or chemical crossflow velocity and $\mathbf{D}$ is the diffusion coefficient.

Introduction

Formula variations

- Most general presentation on steady surfaces defined on Ω as an open domain in $\mathbb{R}^3$ and Γ a connected C^2 compact surface contained in Ω [2].

$$c_t + \mathbf{w} \cdot \nabla_\Gamma c - D \triangle_\Gamma c = 0 \quad \text{on } \Gamma$$

  where $\mathbf{w} : \Omega \to \mathbb{R}^3$ is a divergence-free velocity field in Ω and $\triangle_\Gamma$ denotes the *Laplace-Beltrami* operator on Γ.

- On any planar domain in $\mathbb{R}^3$

$$\frac{\partial C}{\partial t} + \nabla \cdot (\mathbf{u}C) = D\nabla^2 C$$

Outline

2. Preceding studies
   - Mathematical models
   - Conventional finite difference and finite element methods
   - Recent developments for advection–diffusion PDEs

Applied methods I
Towards finite elements methods

The following methods have been widely used progressively to deal with advection and diffusion problems [1].

Miscible flows

A mathematical model used for describing fully saturated fluid flow processes through porous media is derived by using the mass balance equation for the fluid mixture.

Multiphase flows

When either air or a nonaqueous-phase liquid (NAPL) contaminant is present in groundwater transport processes, this phase is immiscible with the water phase and the two phases flow simultaneously in the flow process.

Applied methods I

Dealing with finite elements methods

Finite difference methods (FDMs)

Due to their simplicity, FDMs were first used in solving advection-dominated PDEs.

Galerkin and Petrov–Galerkin Finite element methods (FEMs)

Applied methods I
Beyond Finite Elements Methods

Advection-Diffusion PDEs

- Eulerian methods for advection–diffusion PDEs.
- The streamline diffusion finite element method (SDFEM)
- Total variation diminishing (TVD) methods.
- Essentially nonoscillatory (ENO) schemes and weighted essentially nonoscillatory (WENO) schemes.
- The discontinuous Galerkin (DG) method
- Characteristic methods.

Applied methods II

Beyond Finite Elements Methods

- Classical characteristic or Eulerian–Lagrangian methods.
- The modified method of characteristics (MMOC).
- The modified method of characteristics with adjusted advection (MMOCAA).
- The Eulerian–Lagrangian localized adjoint method (ELLAM).
- The characteristic mixed finite element method (CMFEM).
- Characteristic methods for immiscible fluid flows, operator splitting techniques.

Outline

General and specific objectives summary

- Understand and obtain conceptual and theorical knowledge about the problem.
- Implement the studied method: *Finite Elements Method* variation.
- Build the advection-diffusion partial differential equation weak form in order to apply the numerical method.
- Succeed with the computational implementation of such weak form using the method.

Acknowledgment

THANK YOU FOR YOUR ATTENTION!

QUESTIONS?

Bibliography

[1] Richard E. Ewing and Hong Wang, *A summary of numerical methods for time-dependent advection-dominated partial differential equations*. Journal of Computational and Applied Mathematics (2001): 423 - 445.

[2] Olshanskii, Maxim A. and Reusken, Arnold and Xu, Xianmin, *A stabilized finite element method for advection–diffusion equations on surfaces*. IMA Journal of Numerical Analysis (2013).