Real Options Valuation for mining projects using a proposed numerical schema based on Finite Difference Method

Cuscagua-Lopez, J. Mauricio   Marín-Sanchez, Fredy H.

Research Practice I: Proposal Presentation

Mathematical Engineering

August, 2015

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Content

Some Definitions

- **Option:** contract which gives the buyer - the owner or holder - the right, but not the obligation, to buy or sell an underlying asset or instrument at a specified strike price on or before a specified date.
- **Real Option:** contract which gives the buyer the right — but not the obligation — to undertake certain business initiatives

Why to use Options?

- **Because of its versatility:** They allow for positive movements although the market does not tend to rise.
- **To ensure investment:** Minimize the risk and losses (not eliminate).

Real Option and Financial Option Equivalence

Real Options	Financial Options
Present value of the expected cash flow on time t	The Underlying price at the time t
Investment cost	Strike Price
Free interest rate risk	Free interest rate risk
Volatility of the project's flow cash	Volatility of the Underlying
Time the investment option dissapears	Maturation Time

Taken from [González-Echeverri et al., 2015]

Options Valuation

Let f be the price of an option. To calculate its price the following Partial Differential Equation must be solved [Black and Scholes, 1973]:

$$\frac{\partial f}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 f}{\partial S^2} + rS\frac{\partial f}{\partial S} = rf \quad (1)$$

where f is the price of the Option, S is the price of the underlying, σ is the volatility of the underlying and r is the free interest rate risk.

Note

The boundary conditions depends on the Option's dynamic.

Assumptions

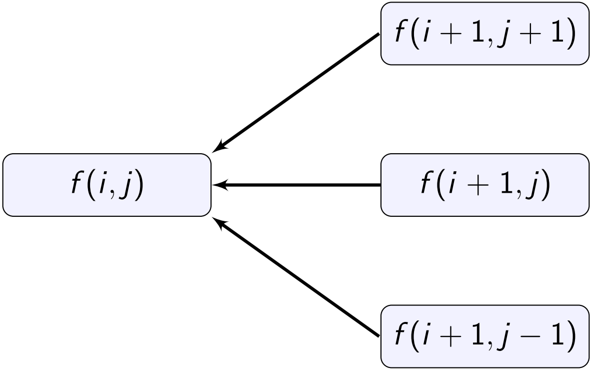
- The price of the underlying is a Geometric Brownian movement
- No transaction costs.
- The assets are perfectly divisible.
- The underlying pays no dividends during the life of the option.
- No arbitrage opportunities.
- The negotiation of assets is continuing.
- Free interest rate risk r is constant for all maturities.

Financial and Real Option Valuation

To solve Equation (1) it's required a numerical method. Finite Difference Method will be implemented to solve it.

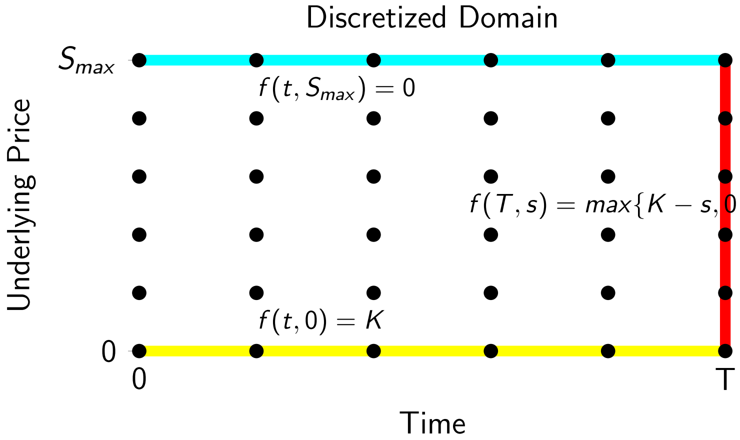
Analogically, we can calculate a Real Option value by numerical methods if the Partial Differential Equation is given.

Finite Difference Method: Dynamic



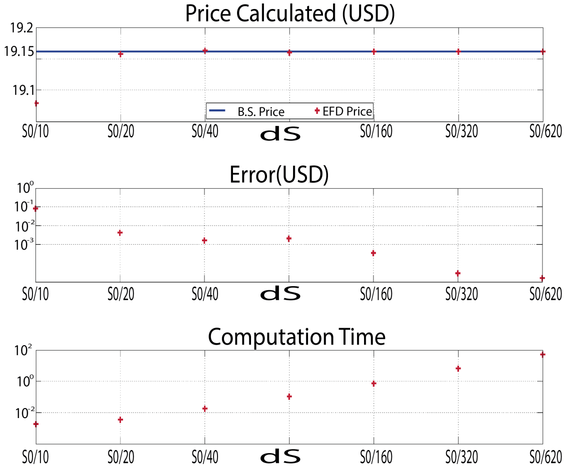
Taken from [Velásquez et al., 2015]

Finite Difference Method: Dynamic



Taken from [Velásquez et al., 2015]

Finite Difference Method: Problems, $dS \rightarrow 0$



Taken from [Velásquez et al., 2015]

Option valuation

- [Schwartz, 1977]: Finite Difference method:
  - Set the boundary conditions when time to excersise the option is over, when the price is maximum and when the price is minimun.
  - Values at other lattice points are calculated recursively begining from the maturity time and ending at time 0.
- [Hull, 2006]: Binomial Tree:
  - Divide the life of the option into many small tame intervals of length δ_t.
  - The price moves from its initial value of S to one of two new values S_u and S_d with given probabilities.

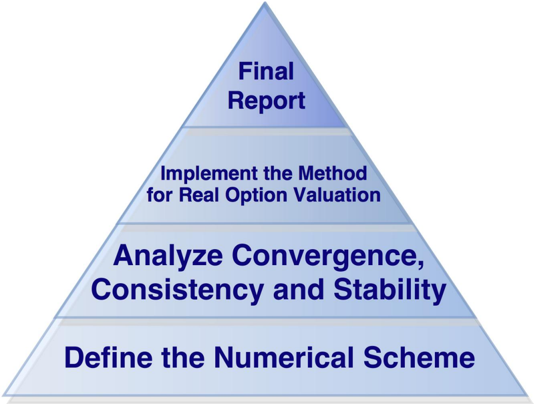
Option valuation

- [Lazo et al., 2009]: Monte Carlo simulation:
  - A backward induction process is performed.
  - The value is recursively assigned.
  - The value of the option is calculated by moving through the time steps and states.

Real option valuation: Mining projects

- [Haque et al., 2014]: Finite Difference method:
  - Given the PDE for a closed mine, Finite Difference method is implemented to sample the PDE.
- [Colwell et al., 2002]: Finite Difference method:
  - Diferent models are describe to valuate the option to open, close and abandon a mine. Given the historical price of gold, Finite Difference method is implemented for mining projects in Australia.

Objectives


Final
Report
Implement the Method
for Real Option Valuation
Analyze Convergence,
Consistency and Stability
Define the Numerical Scheme

Activity	Estimated Time Range
Literature Review	Jul. 21 - Aug. 8
Applying the numerical scheme to Black-Scholes' equation. Make simulations and tests.	Aug. 10 - Aug. 21
Determine the convergence, consistency and stability of the numerical schema	Aug. 21 - Oct. 3
Applying the numerical scheme to specifics PDE from the literature of interest. Make simulations and tests.	Oct. 3 - Oct. 15
Extra work.	Oct. 16 - Nov. 3

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