Vessel Extraction Using the Buckmaster-Airy Filter

Progress presentation
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Objectives

Design a technique for vessel extraction from biomedical images using a determinated filter.

- Develop a filter for biomedical images processing using partial differential equations.
- Compare the performance of the Buckmaster-Airy filter with other filters obtained from partial differential equations.
- Present the results of the project in an international conference.

Method

Use the two dimensional Airy diffusion equation, which serves to describe linear dispersion in terms of a third-order partial differential equation, and the two dimensional Buckmaster equation, which describes thin viscous fluid sheet flow.

One-dimensional Airy equation

$$\frac{\partial}{\partial t} P(x,t) = \eta \left(\frac{\partial^3}{\partial x^3} P(x,t) \right) \quad (1)$$

with the initial condition

$$P(x,0) = \delta(x - X) \quad (2)$$

One-dimensional Airy equation

Taking the Fourier transform respect to x we have that

$$\frac{\partial}{\partial t}\mathcal{F}\{P(x,t)\} = -i\eta k^3 \mathcal{F}\{P(x,t)\}$$

where

$$\mathcal{F}\{P(x,t)\} = \frac{1}{2}\sqrt{\frac{2}{\pi}}\int_{-\infty}^{\infty} P(x,t)\, e^{-ikx} dx$$

One-dimensional Airy equation

Making the substitution $\mathcal{F}\{P(x,t)\} = P(t)$, equations (1) and (2) adopt the forms

$$\frac{\partial}{\partial t}P(t) = -i\eta k^3 P(t) \qquad (3)$$

$$P(0) = e^{-iXk} \qquad (4)$$

One-dimensional Airy equation

Then solving (3) with the initial condition (4) we have

$$P(t) = e^{-iXk}e^{-i\eta k^3 t} \quad (5)$$

The inverse Fourier transform of (5) can be rewritten as

$$P(x,t) = \frac{3^{2/3} Ai\left(\frac{3^{2/3}(-x+X)}{3\sigma}\right)}{3\sigma} \quad (6)$$

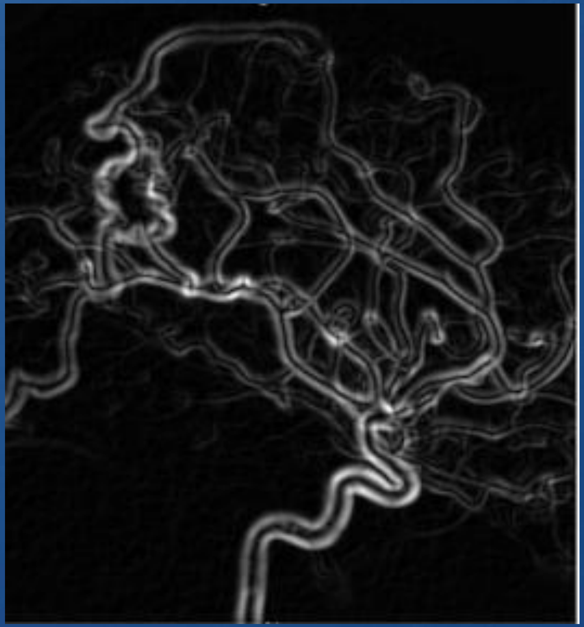
where $\sigma^3 = \eta t$.

Two-dimensional Airy equation

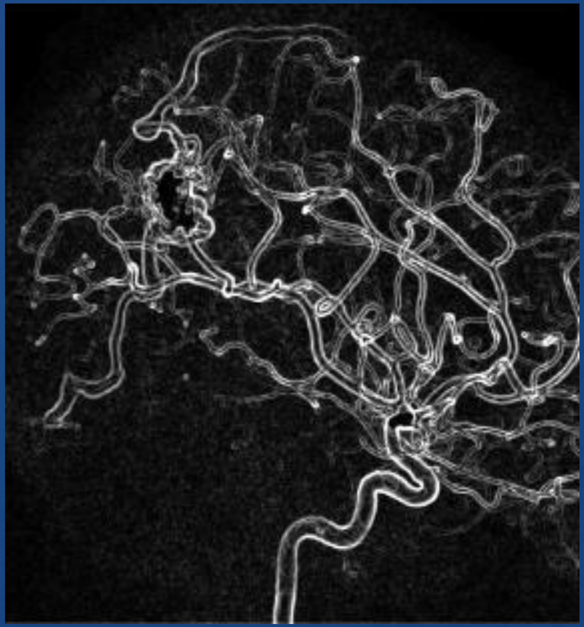
Similarly, the solution for the two-dimensional Airy equation can be found as

$$P(x,y,t) = \frac{3^{1/3} Ai\left(\frac{3^{2/3}(-x+X)}{3\sigma}\right) Ai\left(\frac{3^{2/3}(-y+Y)}{3\sigma}\right)}{3\sigma^2} \quad (7)$$

Edge detection



Using Airy filter



Using Sobel filter

Original image taken from The Aneurysm and AVM Foudation: http://www.taafonline.org/am_detection.html

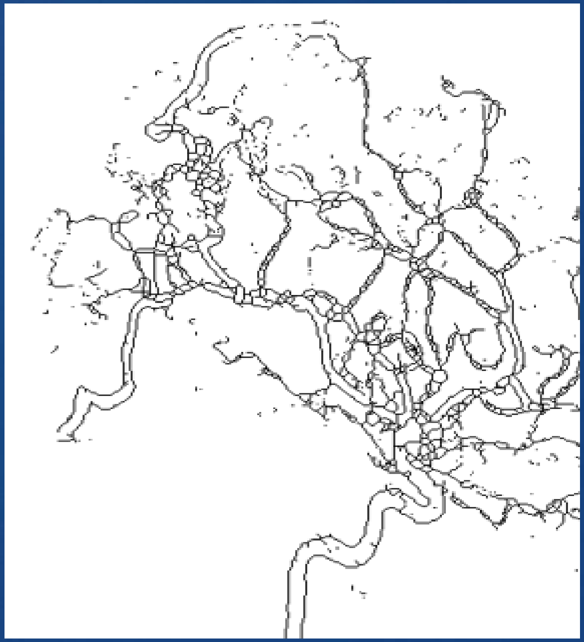
Cany edge detection

Using Airy filter

Using Sobel filter

Skeletonization

Using Airy filter



Using Sobel filter

Buckmaster operator

$$\frac{\partial}{\partial t}u(x,y,t) = \left(\frac{\partial^2}{\partial x^2}u(x,y,t)^4\right) + \left(\frac{\partial}{\partial x}u(x,y,t)^3\right)$$
$$+ \left(\frac{\partial^2}{\partial y^2}u(x,y,t)^4\right) + \left(\frac{\partial}{\partial y}u(x,y,t)^3\right)$$

According to the last equation [1], we construct a filter named here the Buckmaster-Airy filter applying the spatial operator Buckmaster to the solution (7).

Buckmaster-Airy function

$$BA(x,y) = \frac{8}{27\sigma^{10}}\left(3^{2/3}Q^4\right) - \frac{18}{\sigma^7}\left(3^{2/3}Q^3\right) + \frac{4\left((-x+X)+(-y+Y)\right)}{81\sigma^{11}}\left(3^{1/3}Q^4\right)$$

where

$$Q = Ai\left(\frac{3^{2/3}(-x+X)}{3\sigma}\right)Ai\left(\frac{3^{2/3}(-y+Y)}{3\sigma}\right)$$

References

[1] E.A. Hussain and Z. M. Alwan, “The Finite Volume Method for solving Buckmaster’s Equation, Fisher’s Equation and Sine Gordon’s Equation for PDE’s”, in *International Mathematical Forum,* vol. 8, pp. 599-627, 2013.

Questions?