Algorithm for the Study of Expected Aggregated Supply Curves in Deregulated Electricity Markets

Research Practise I Progress Presentation

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- $f_1 = f_2 = ... = f_N = f$; (symmetrical) Nash equilibrium.

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- Identify and make improvements for the algorithm.
- Adapt the algorithm in order to run it using parallel computing.

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where

$$\pi_i = P(d = i)$$

$$H_i = \binom{N-1}{i-1} F(p)^k (1 - F(p))^{N-i}$$

$$G_i(p) = \sum_{k=i-1}^{N-1} \binom{N-1}{k} F(p)^k (1 - F(p))^{N-1-k}$$

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$$f(x) = \alpha exp_q(\beta x) + \gamma = \alpha[1 + (1-q)\beta x]_+^{\frac{1}{1-q}} + \gamma$$

with

$$\alpha, \beta, \gamma, q > 0 \quad \text{and} \quad N - \frac{1}{2} < \frac{1}{\beta(q-1)}$$

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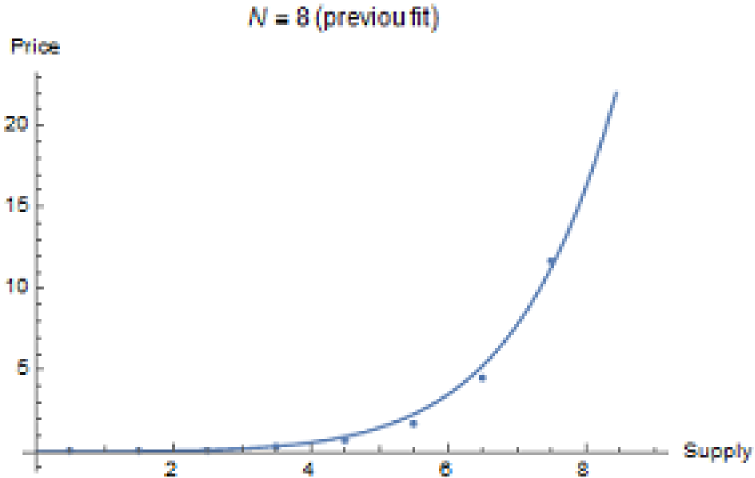

Figure 1: Fitted curve using a previous algorithm

Results

Fit

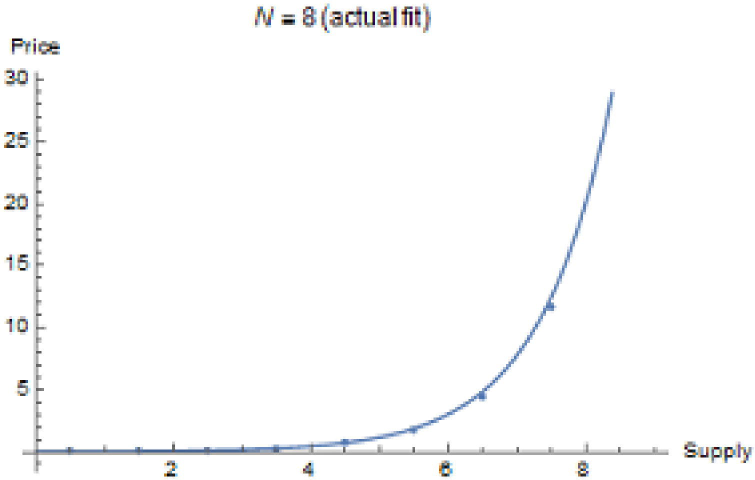

Figure 2: Fitted curve using Mathematica's function FindMinimum

Table 1: Errors of fitting

N	Previous fit SSE	Actual fit SSE
5	290.59	0.01
6	265.47	0.02
7	226.25	0.05
8	181.55	0.18
9	148.17	0.03
10	119.14	0.02
11	94.95	0.01
12	80.38	0.01
13	66.61	0.02
14	55.69	0.02
15	46.67	0.01

Table 2: Execution time

N	t-Normal (Sec)	t-Parallelized (Sec)
5	2.0194	1.83669
6	2.51662	2.17846
7	3.31717	2.62061
8	5.24162	3.37762
9	9.87036	4.72343
10	21.7877	8.01451
11	52.5736	17.5051
12	131.711	40.8619
13	325.653	86.8948
14	813.039	233.064
15	2177.97	619.596

Table 3: Values of parameters

N	α	β	q
5	9.3	0.56	0.78
6	3.98	0.64	0.84
7	1.71	0.69	0.88
8	0.71	0.75	0.90
9	0.32	0.77	0.92
10	0.13	0.80	0.93
11	0.05	0.86	0.93
12	0.006	0.82	0.95
13	0.01	0.86	0.95
14	0.005	0.83	0.96
15	0.002	0.91	0.95

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Bibliography

[1] von der Fehr, N-H. M., Harbord, D., *Spot Market Competition in the UK Electricity Industry*. The Economic Journal (1993): 531-546.