Brain Tumor Detection Using the Ricci Tensor, Anisotropic Diffusion and the Canny Filter

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Research practice I: progress report
October 16, 2015

Content

Airy Filter

The two-dimensional Airy equation that softens the images has this form

$$\frac{\partial}{\partial t}P(x,y,t) = \eta_1\left(\frac{\partial^3}{\partial x^3}P(x,y,t)\right) + \eta_2\left(\frac{\partial^3}{\partial y^3}P(x,y,t)\right) \qquad (1)$$

with the initial condition

$$P(x,y,0) = Dirac(x-X)Dirac(y-Y)$$

Airy Filter

and with the change of variable

$$\eta = \frac{\sigma^3}{t}$$

Solution of (1) takes the form

$$P(x, y, \sigma) = \left(\frac{1}{3}\right) \frac{3^{\frac{1}{3}}\, AiryAi\left(\frac{3^{\frac{2}{3}}(-x+X)}{3\sigma}\right) AiryAi\left(\frac{3^{\frac{2}{3}}(-y+Y)}{3\sigma}\right)}{\sigma^2}$$

Riemann tensor

The metric tensor defined according to

$$ds^2 = \sum_j^n \left(\sum_i^n g_{ij} dx^i dx^j \right)$$

with ds^2 defining the differential of length.

- From the Riemann tensor is possible to define the Ricci tensor as

$$R_{ij} = R^k_{ikj} \qquad \forall k = 1, ..., n$$

Ricci tensor

The Ricci tensor for a two-dimensional surface defined by $z = f(x, y)$, is given by [1]

$$\begin{bmatrix} R_{xx} & R_{xy} \\ R_{xy} & R_{yy} \end{bmatrix}$$

These expressions for the Ricci tensor correspond to a metric tensor of the form [4]

$$g_{ij} = \begin{bmatrix} 1 + (\frac{\partial}{\partial x} f(x, y))^2 & (\frac{\partial}{\partial x} f(x, y))(\frac{\partial}{\partial y} f(x, y)) \\ (\frac{\partial}{\partial x} f(x, y))(\frac{\partial}{\partial y} f(x, y)) & 1 + (\frac{\partial}{\partial y} f(x, y))^2 \end{bmatrix}$$

Ricci tensor

Associated by a line element given by

$$ds^2 = dx^2 + dy^2 + dz^2$$

with $z = f(x, y)$

$$ds^2 = \begin{bmatrix} dx & dy \end{bmatrix} \begin{bmatrix} 1 + (\frac{\partial}{\partial x} f(x,y))^2 & (\frac{\partial}{\partial x} f(x,y))(\frac{\partial}{\partial y} f(x,y)) \\ (\frac{\partial}{\partial x} f(x,y))(\frac{\partial}{\partial y} f(x,y)) & 1 + (\frac{\partial}{\partial y} f(x,y))^2 \end{bmatrix} \begin{bmatrix} dx \\ dy \end{bmatrix}$$

Anisotropic Diffusion

The anisotropic diffusion equation has the form

$$\frac{\partial I}{\partial t} = div(c(x,y,t) \bigtriangledown I) = \bigtriangledown c \bullet \bigtriangledown I + c(x,y,t) \bigtriangleup I$$

$$c(\| \bigtriangledown I \|) = e^{-(\frac{\bigtriangledown I}{k})^2}$$

$$c(\| \bigtriangledown I \|) = \frac{1}{1 + (\frac{\bigtriangledown I}{k})^2}$$

where

div is divergence operator and $\bigtriangledown I$ is gradient operator.

Anisotropic Diffusion - Results

In Figure we observe the result from the application of anisotropic diffusion to the image in right Figure. The experiment was performed using the package ImageJ.

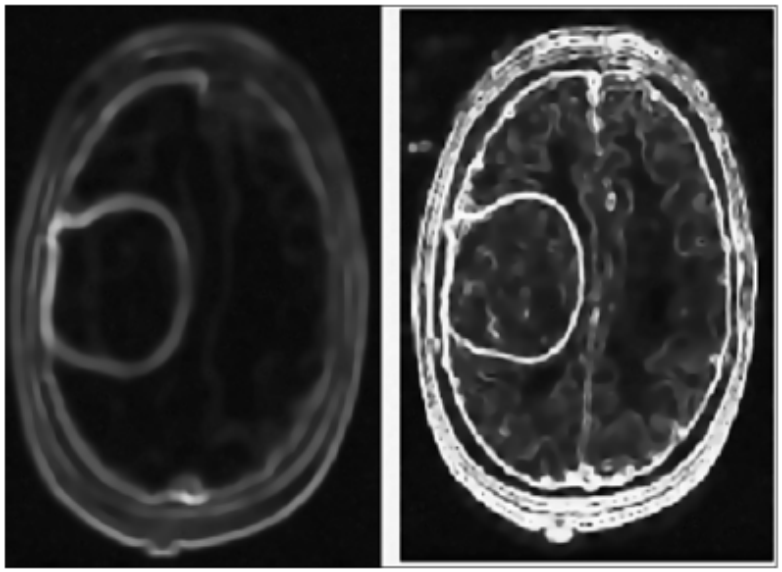

FIGURE: Results of the application of anisotropic diffusion [1].

Canny Edge Detector

Two experiments with the Canny edge detector were performed.

- The result is shown in the following Figure.
- These two experiments with the Canny edge detector were performed using the program ImageJ.

Canny Edge Detector

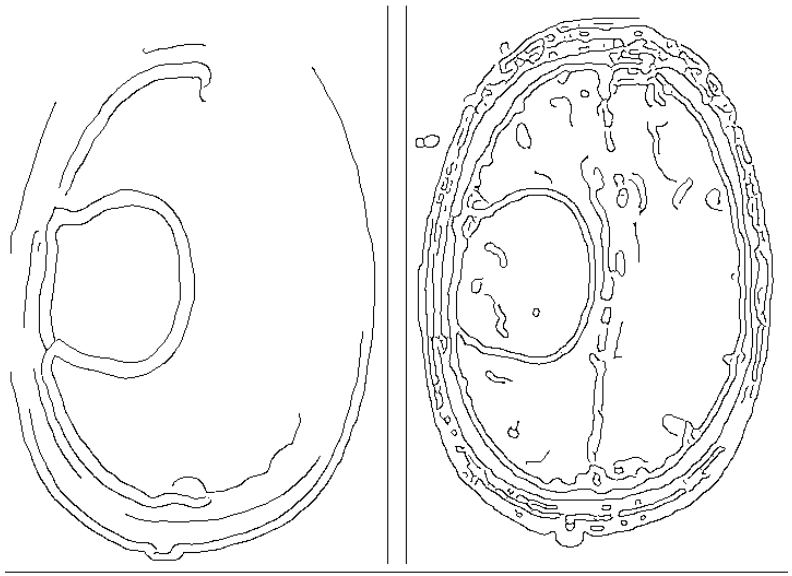

FIGURE: The result from the application of the Canny edge detector to the image in right Figure with anisotropic diffusion with 100 iterations.

Result

The Ricci tensor image is processed using anisotropic diffusion and then resulting image is processed with the Canny edge detector. The anisotropic diffusion and the Canny filter are applied using the program ImageJ

Results of the first experiment

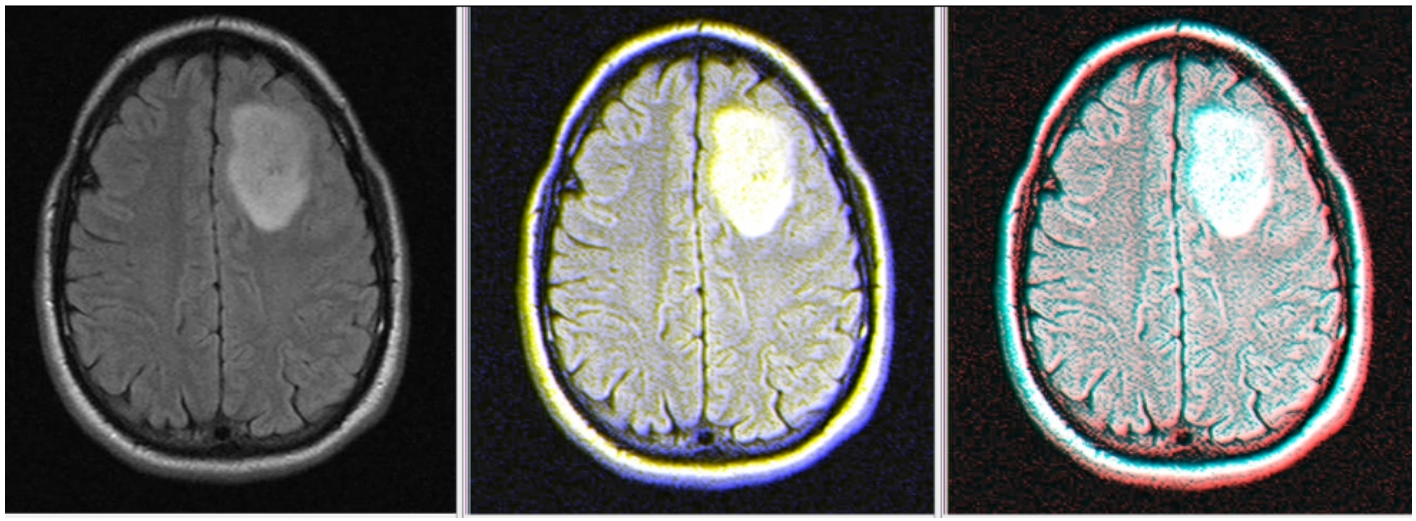

FIGURE: Results of the first experiment with the RicciTensor-Airy filter and Canny edge detector.

Results of the first experiment

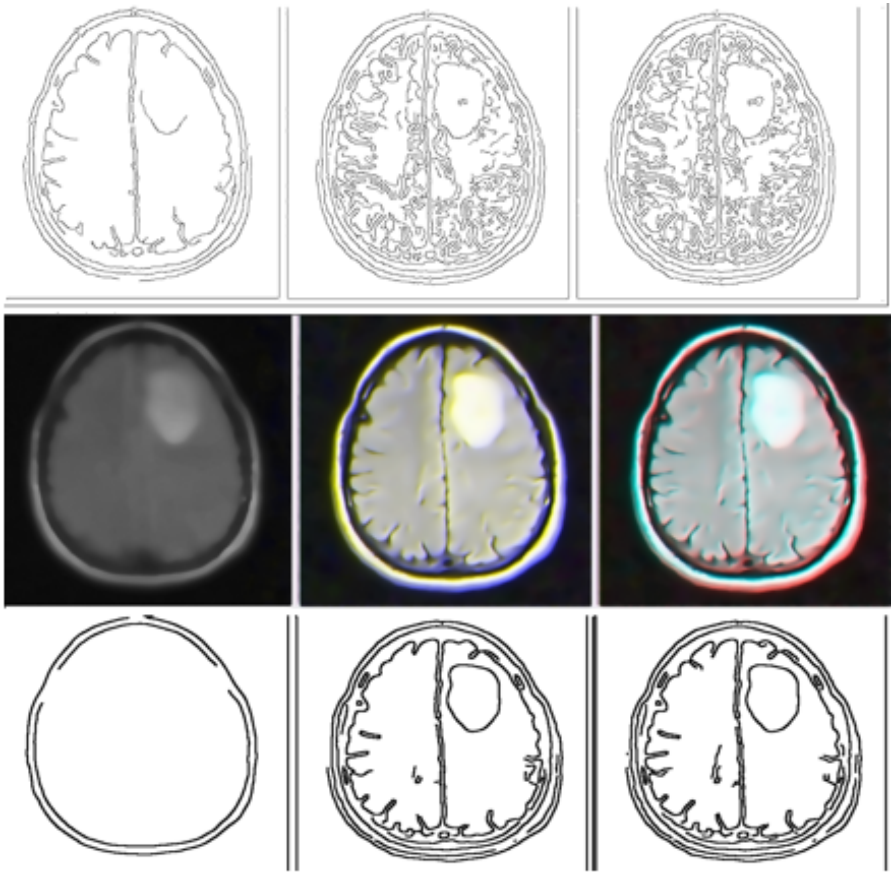

FIGURE: Results of the first experiment with anisotropic diffusion and Canny edge detector.

Results of a second experiment

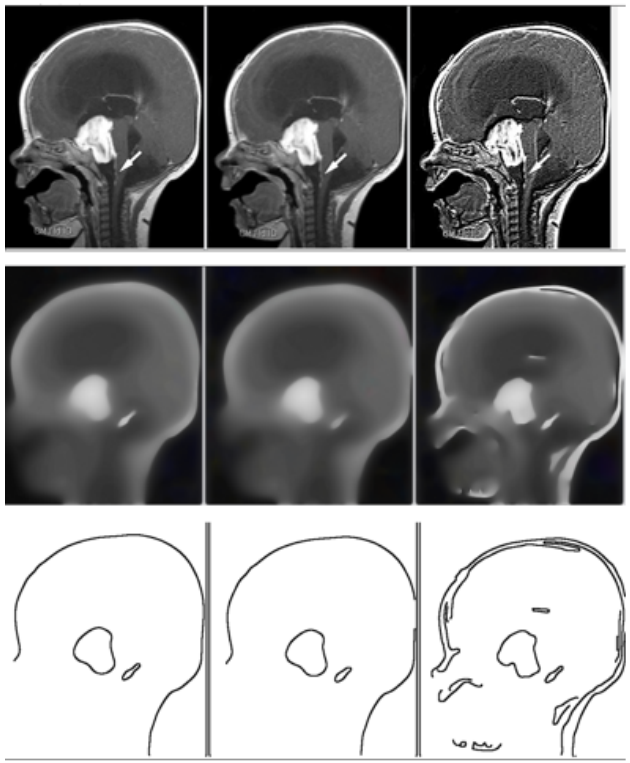

FIGURE: Results of a second experiment with RicciTensor-Airy AnisotroDiff and Canny.

Conclusions

- Some experiments of brain tumor detection were performed and the results were excellent. The combination of the Airy-Ricci filter with anisotropic diffusion and the Canny edge detector is able to produce an enhancement of the tumor in the image without producing alterations on the edges of the tumor.

References

1. P. Sharma, M. Diwakar, and S. Choudhary, Application of edge detection for brain tumor detection," International Journal of Computer Applications, vol. 58, no. 16, pp. 2125, 2012.

Thanks for your attention