ANALYSIS OF PROCESSES CAPABILITY USING THE SKEWED NORMAL DISTRIBUTION

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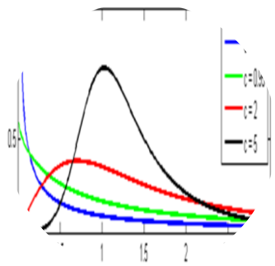
Process capability indices requires normality



In practice there are interest variables that do not follow a normal distribution.

$$C_p = \frac{USL - LSL}{6\,\sigma}$$

Traditional process capability indices are sensitive to non-normality of data.

Inspira Crea Transforma

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Methods to estimate process capability indices associated with non-normal data

Clements's percentile method:

It calculates the indices using a family of Pearson curves [1].

Box-Cox transformation method:

It consists of an initial data transformation followed by the application of conventional methods to resulting data considered as normal [2].

[1] Clements, J. A. (1989). "Process capability indices for non-normal calculations". Quality Progress, 22, 49-55.
[2] Ahmad, S., Abdollahian, M. and Zeephongsekul, P. (2008). "Process capability estimation for non – normal quality characteristics: A comparison of Clements, Burr and Box – Cox Methods". ANZIAM Journal, 49, 642–665.

Clements's percentile method

Index [1]	Normal	Non-Normal
Potential process capability index.	$C_p = \frac{USL - LSL}{6\sigma}$	$C_p = \frac{USL - LSL}{P_{0.99865} - P_{0.00135}}$
Capability index for the lower specification level.	$C_{pl} = \frac{\mu - LSL}{3\sigma}$	$C_{pl} = \frac{P_{0.5} - LSL}{P_{0.5} - P_{0.00135}}$
Capability index for the upper specification level.	$C_{pu} = \frac{USL - \mu}{3\sigma}$	$C_{pu} = \frac{USL - P_{0.5}}{P_{0.99865} - P_{0.5}}$
Real process capability index.	$C_{pk} = min\{C_{pl}, C_{pu}\}$	$C_{pk} = min\{C_{pl}, C_{pu}\}$

Where:
- μ: real mean
- σ^2: real variance
- USL and LSL: upper and lower specification limits
- P_q: q percentile, with $0 < q < 1$

[1] Clements, J. A. (1989). "Process capability indices for non-normal calculations". Quality Progress, 22, 49-55.

Skewed Normal distribution

The probability density function associated to a random variable with a Skewed Normal distribution is as follows [3]:

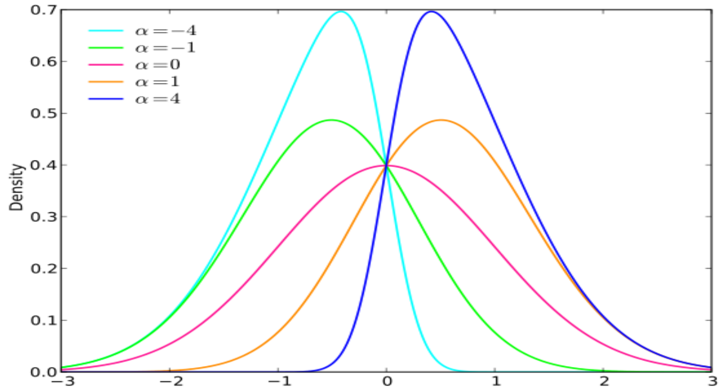
$$f(x) = \frac{1}{\omega\pi} e^{-\frac{(x-\xi)^2}{2\omega^2}} \int_{-\infty}^{\alpha\left(\frac{x-\xi}{\omega}\right)} e^{-\frac{t^2}{2}} \, dt$$

Where:

ξ is a position parameter $\longleftrightarrow \mu$

ω is a scaling parameter $\longleftrightarrow \sigma$

α is a shape parameter



Taken from: www.wikipedia.com

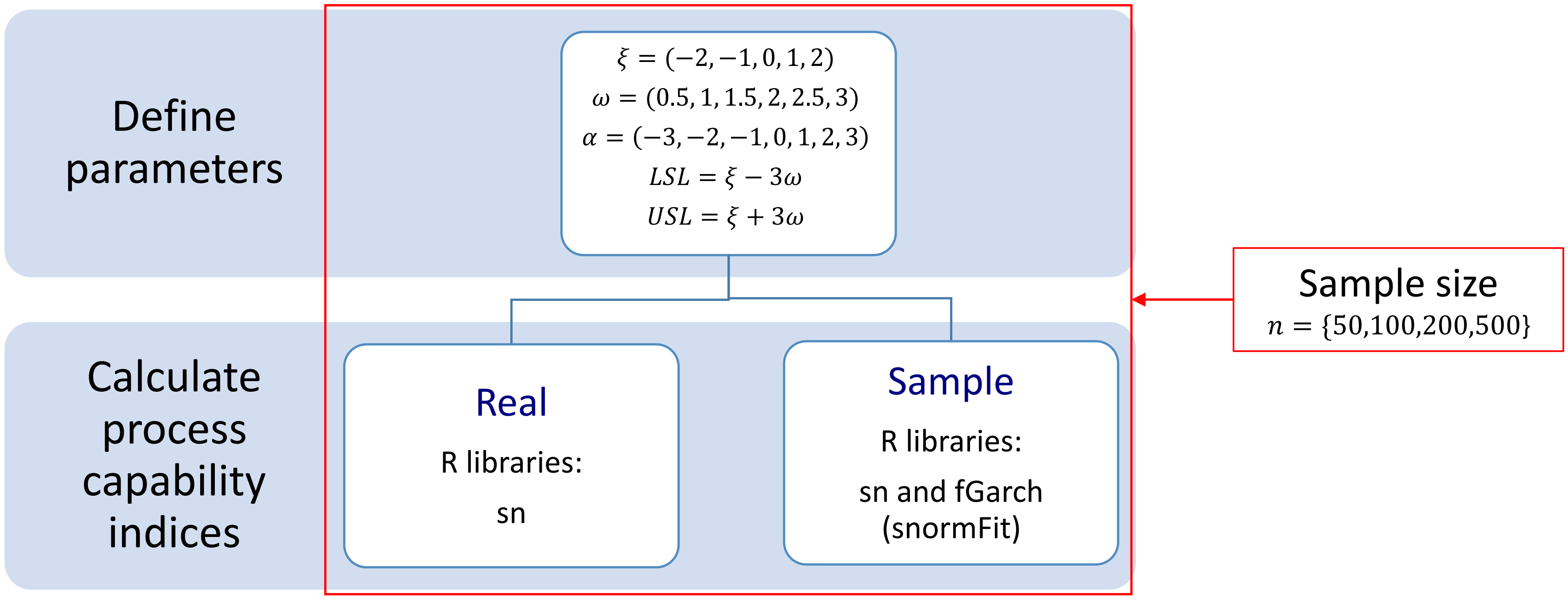
$\alpha > 0 \longrightarrow$ Positive asymmetry

$\alpha = 0 \longrightarrow$ Symmetry

$\alpha < 0 \longrightarrow$ Negative asymmetry

[3] Figueiredo, F. and Gomes, I. (2011) "The skew-normal distribution in SPC". National Funds through Fundação para a Ciência e a Tecnologia.

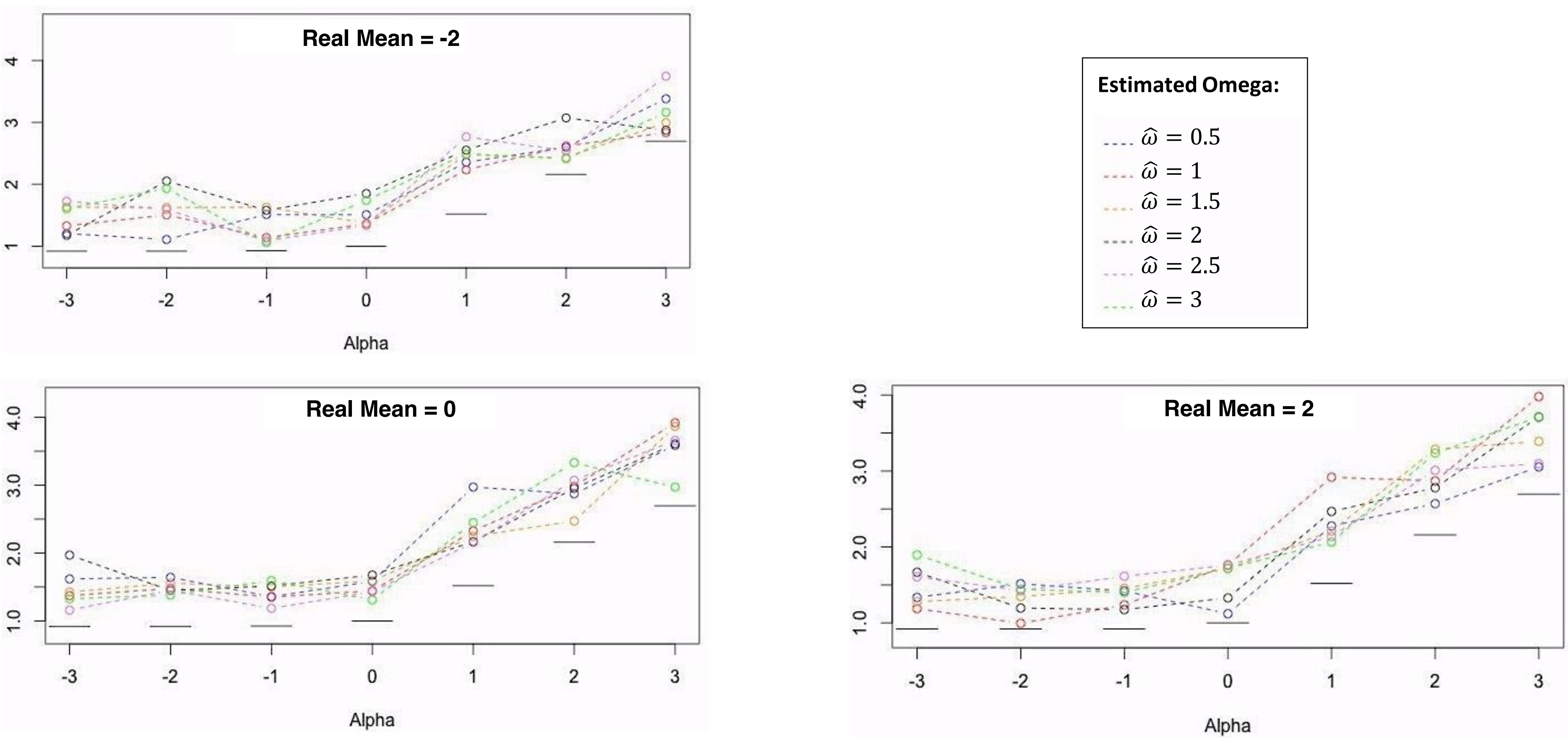
Method adaptation in R



Number of studied scenarios = 840

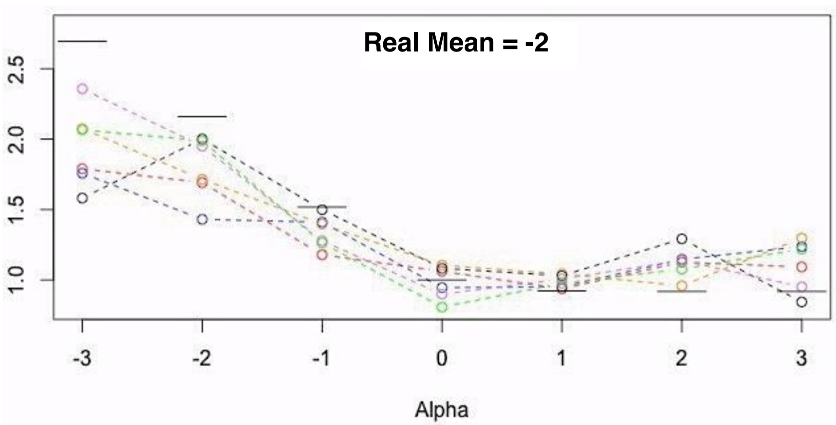
The shape parameter (α) sets a trend in estimates of process capability indices

Index C_{pl} for sample size 50



The shape parameter (α) sets a trend in estimates of process capability indices

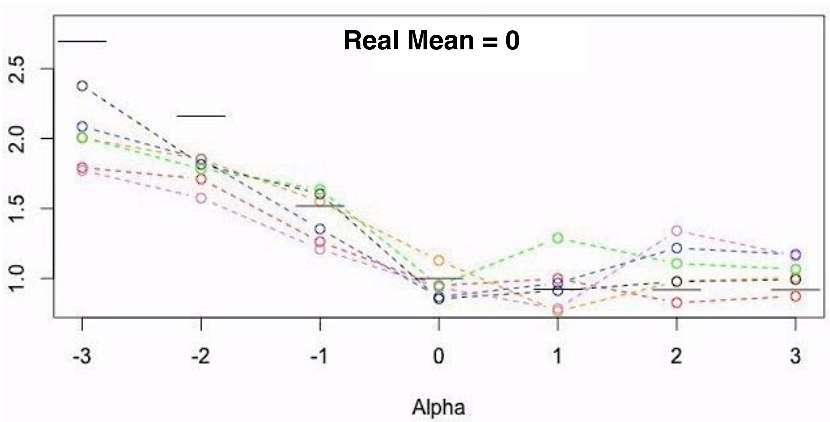
Index C_{pu} for sample size 50

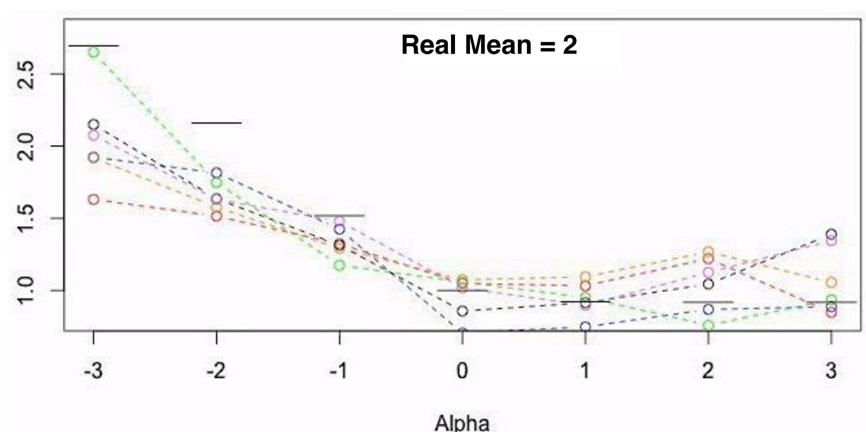


Estimated Omega:

- $\widehat{\omega} = 0.5$
- $\widehat{\omega} = 1$
- $\widehat{\omega} = 1.5$
- $\widehat{\omega} = 2$
- $\widehat{\omega} = 2.5$
- $\widehat{\omega} = 3$

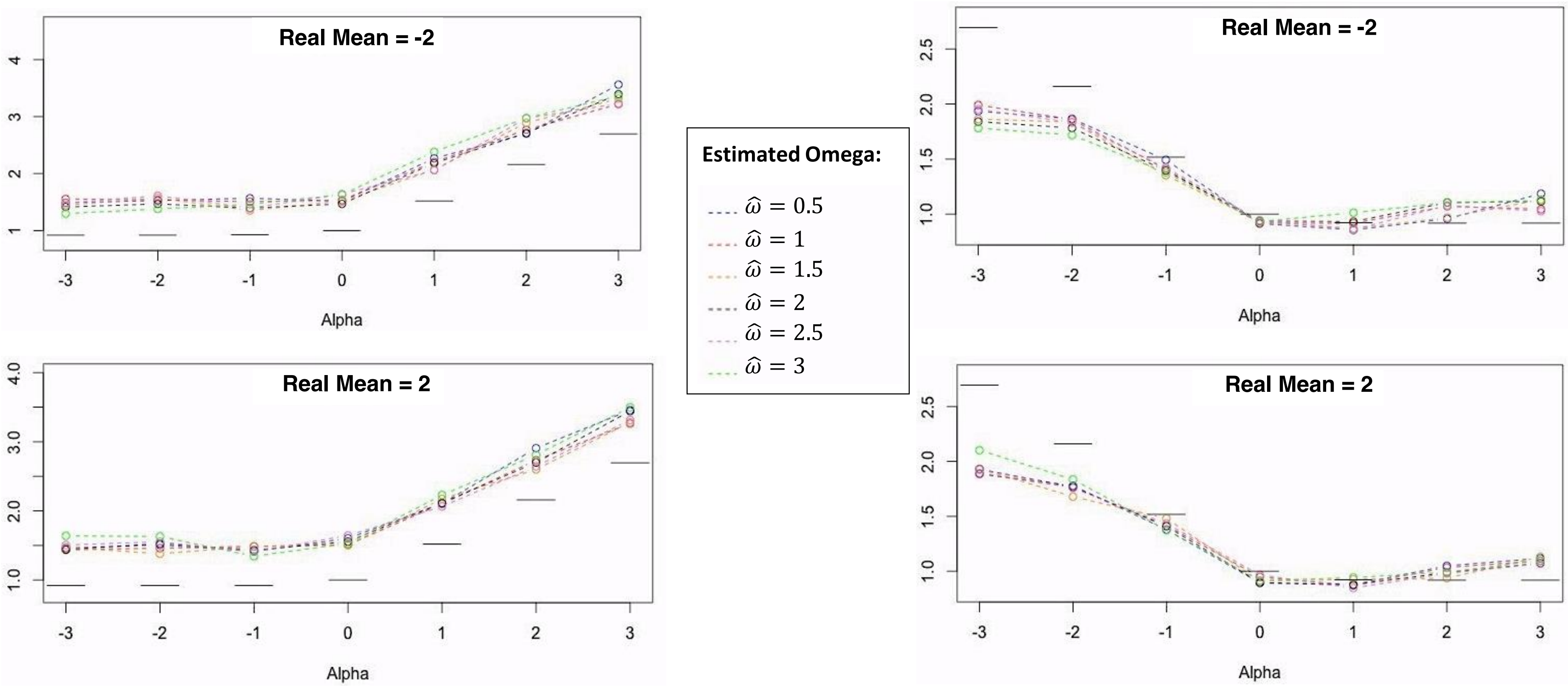
The trend is independent of the values of the parameters of position and scale.



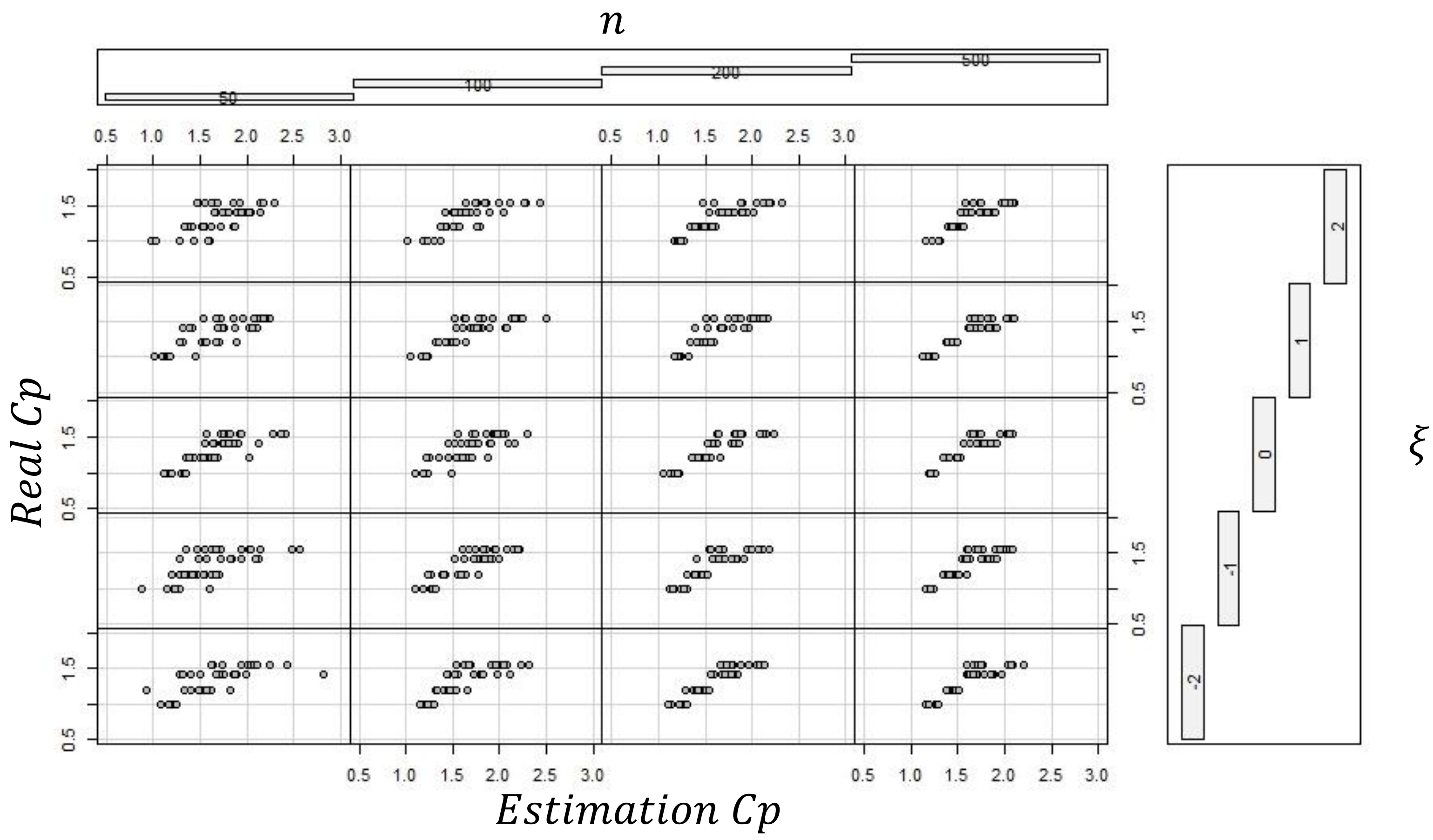


The trend continues for different sample sizes

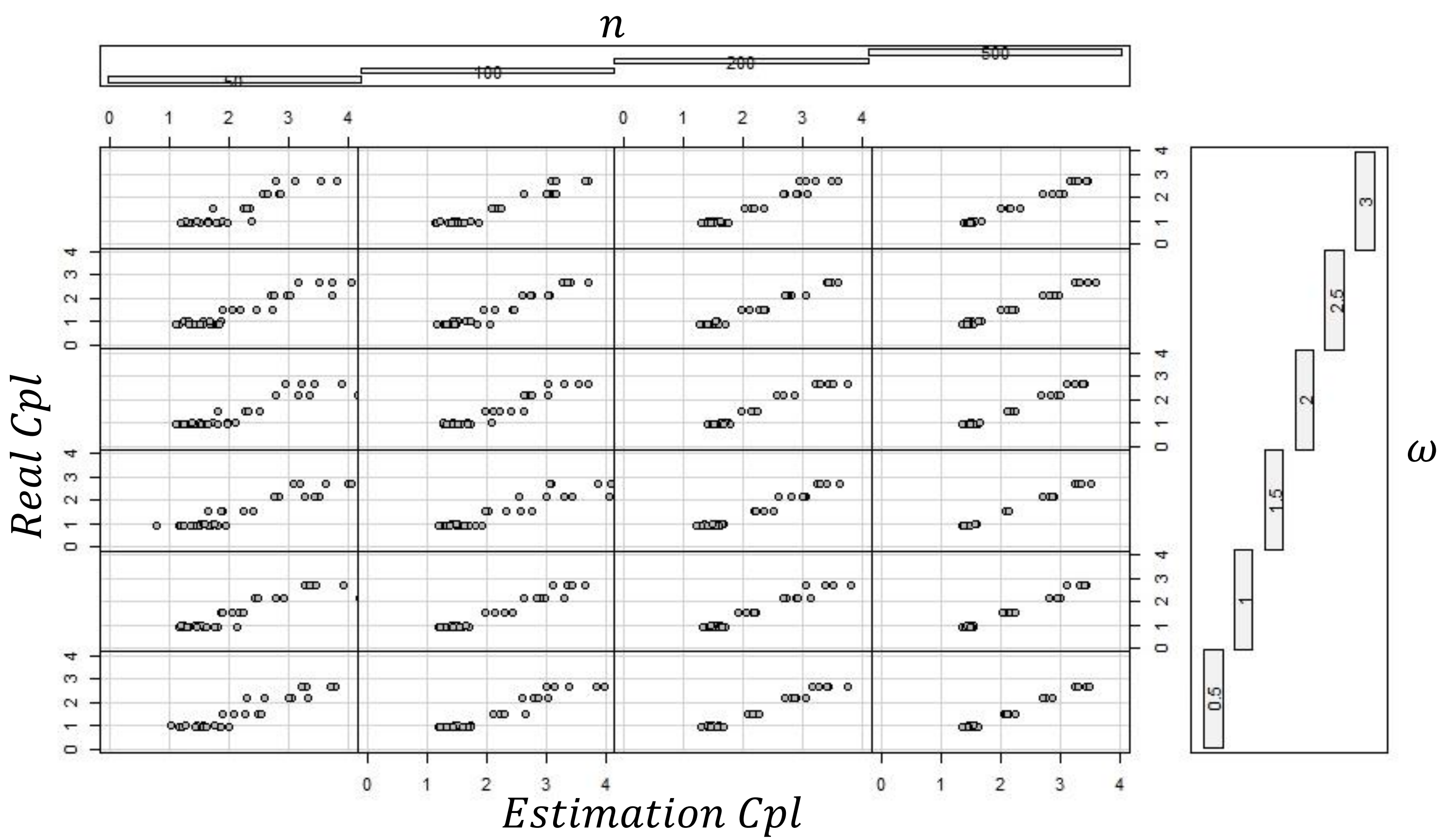
C_{pl} C_{pu}



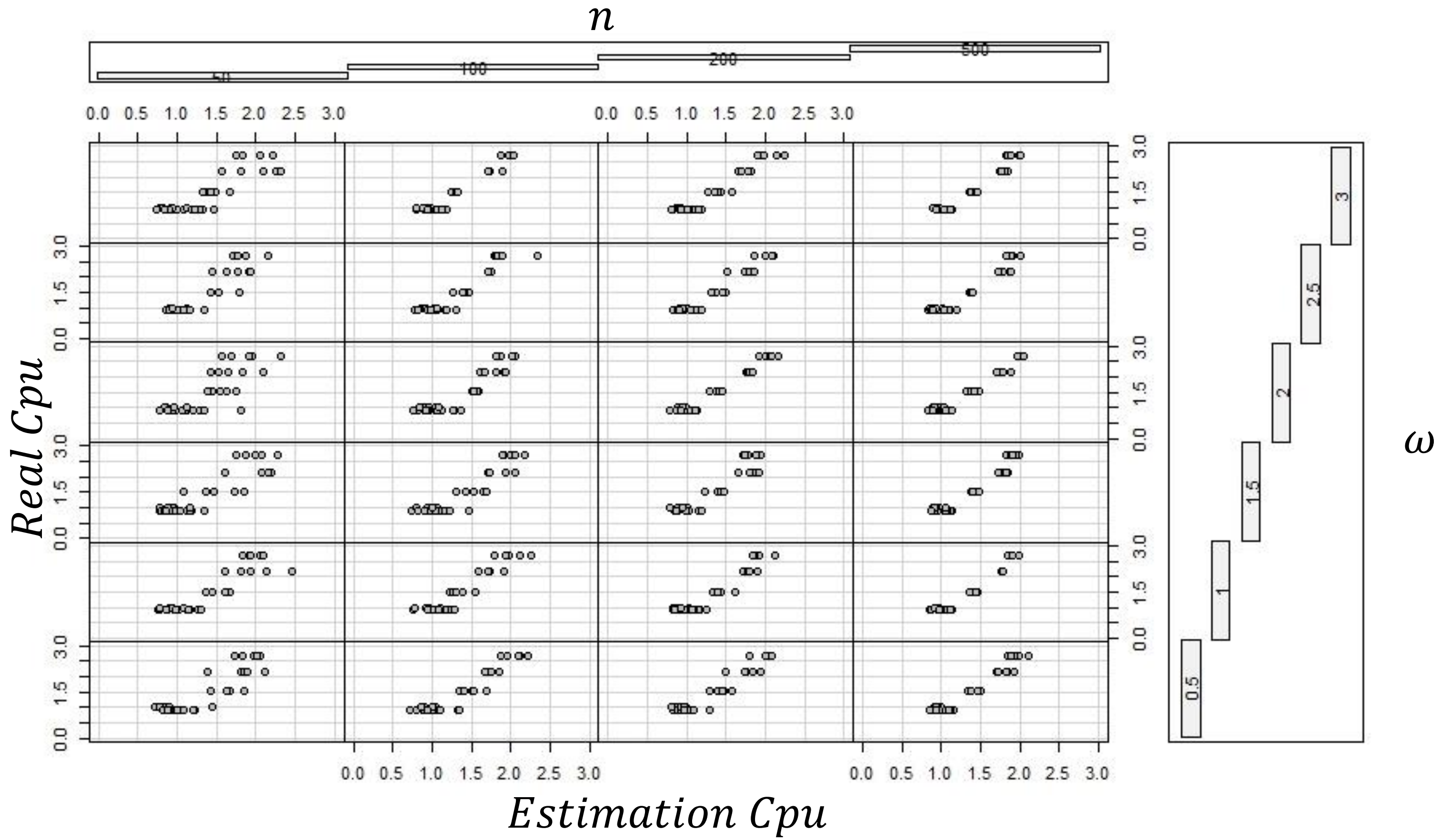
Consistent estimators are obtained for any value of ξ



This consistency is more obvious to the position parameter ω

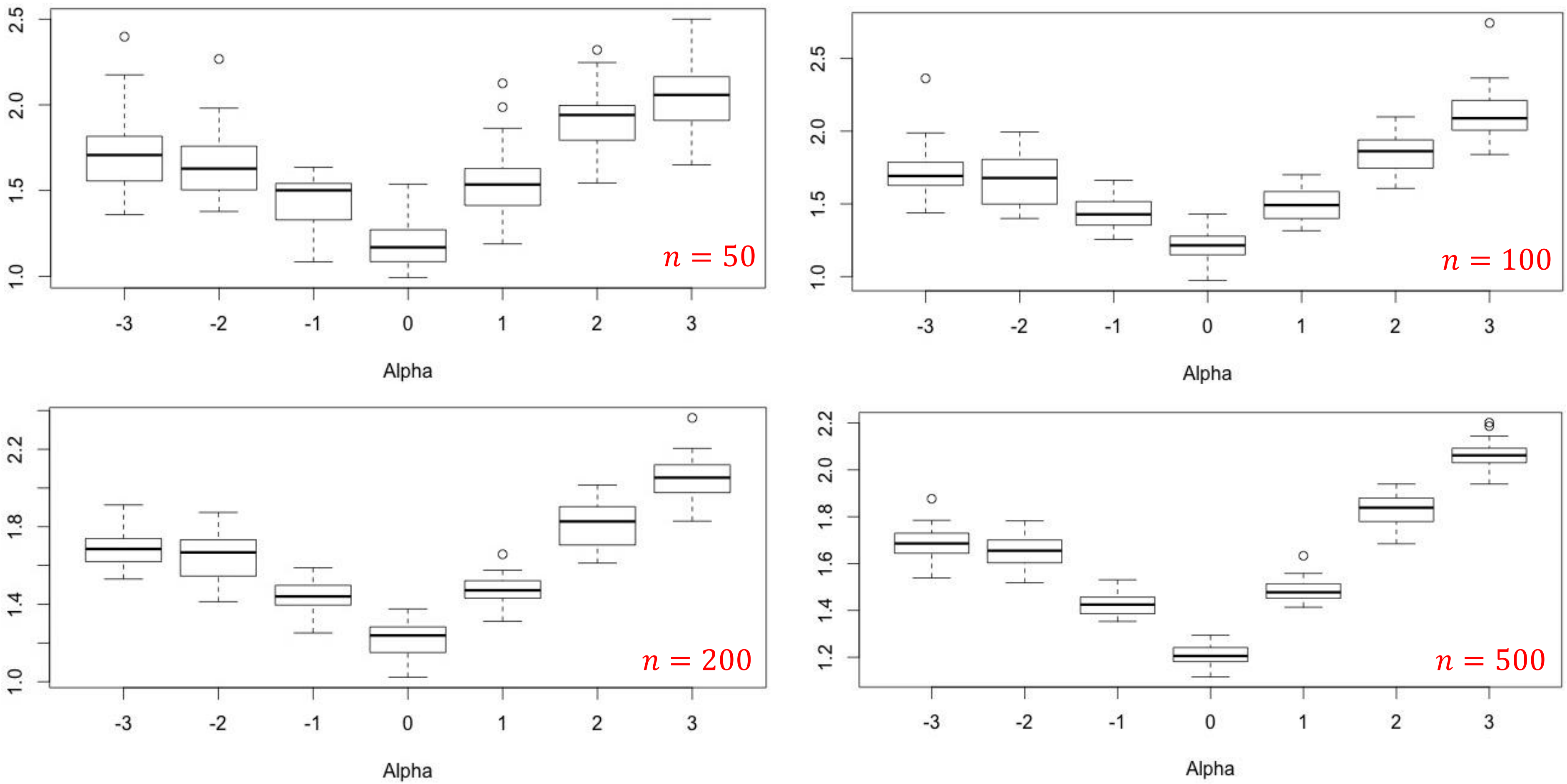


The consistency of ξ and ω is satisfied for any index



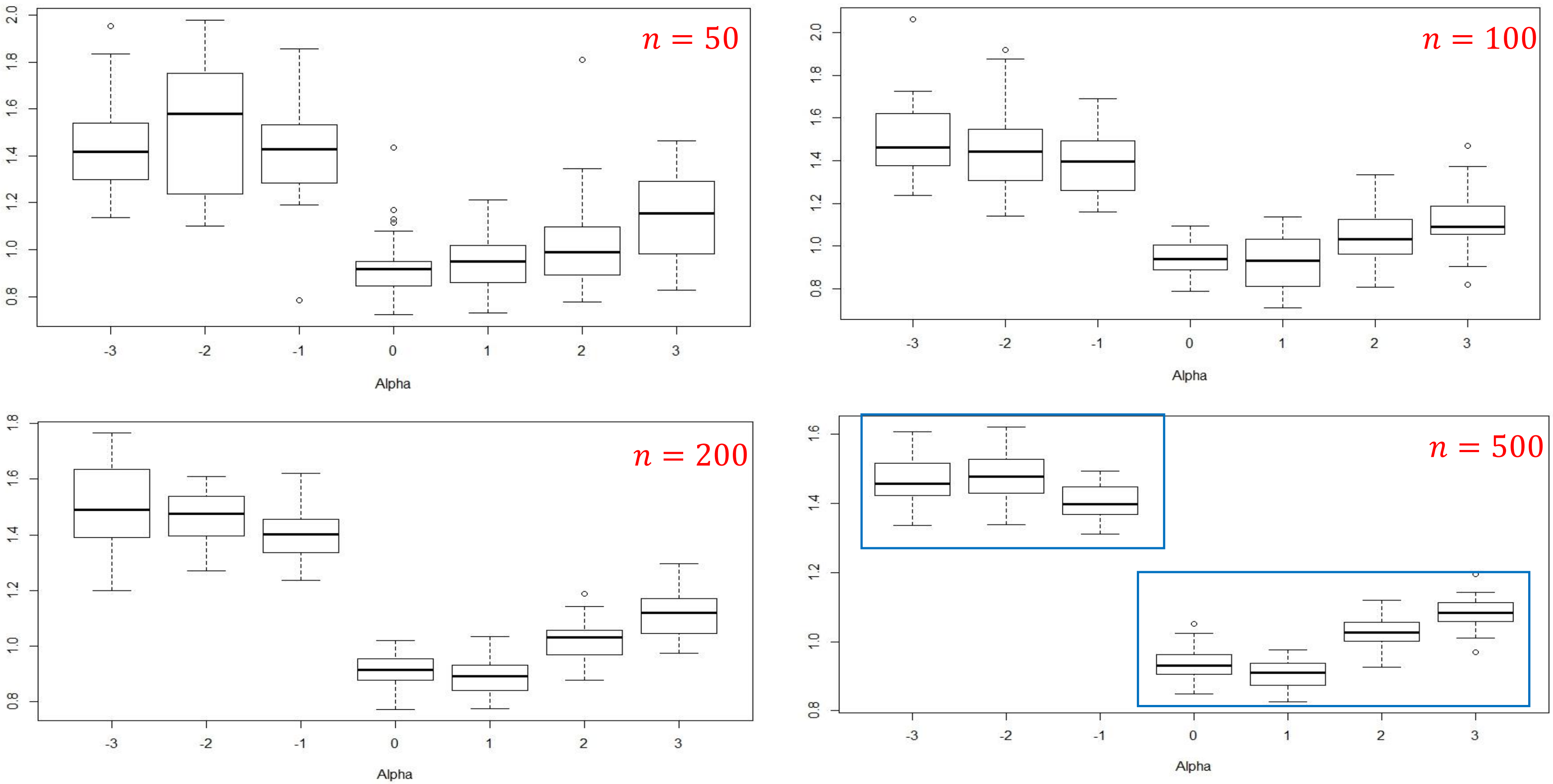
Limits for C_p based on the shape parameter α

There is some symmetric in the index values.



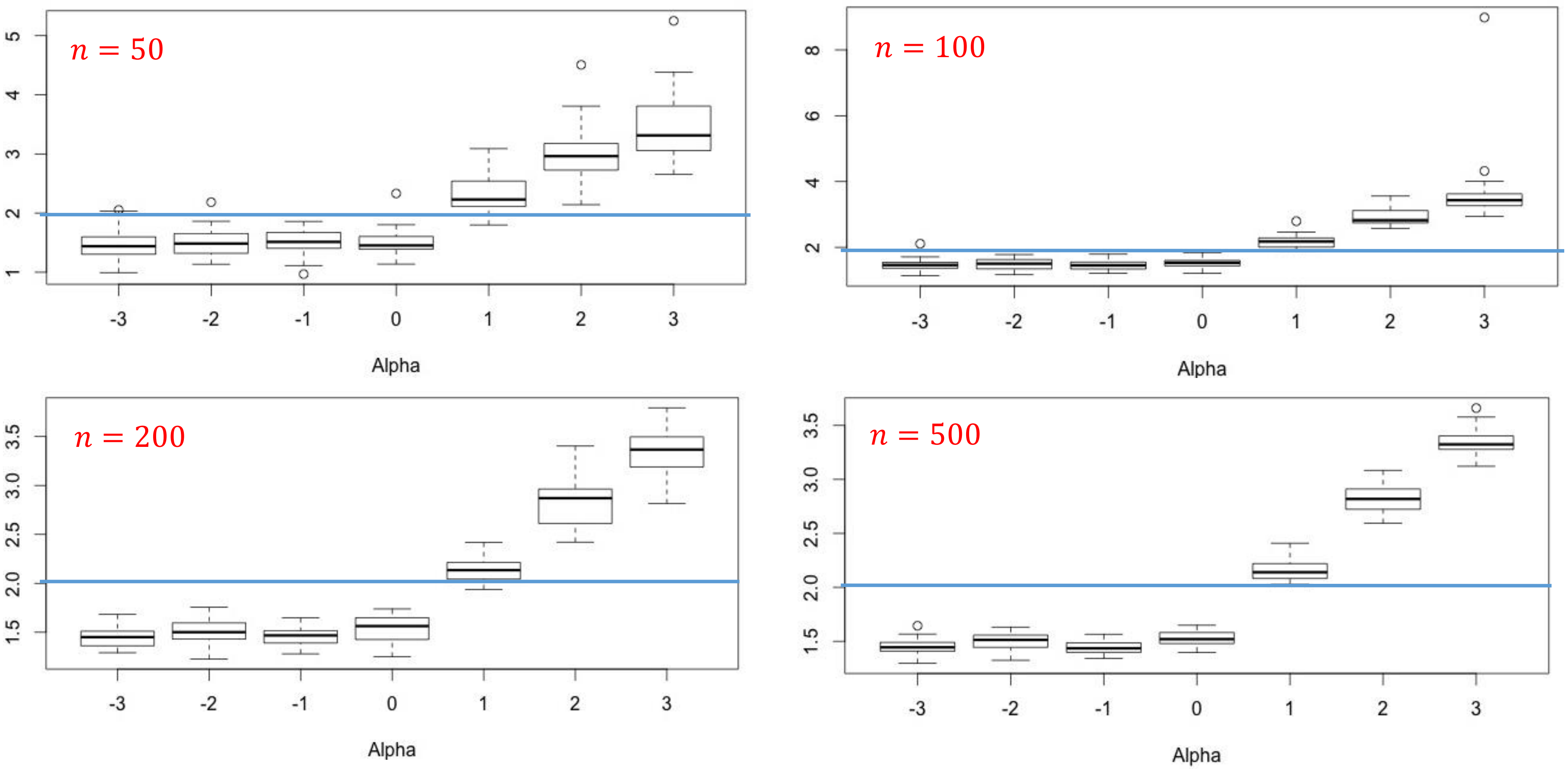
Limits for C_{pk} based on the shape parameter α

Two sets of values are established.



Limits for C_{pl} based on the shape parameter α

There is a cutoff point

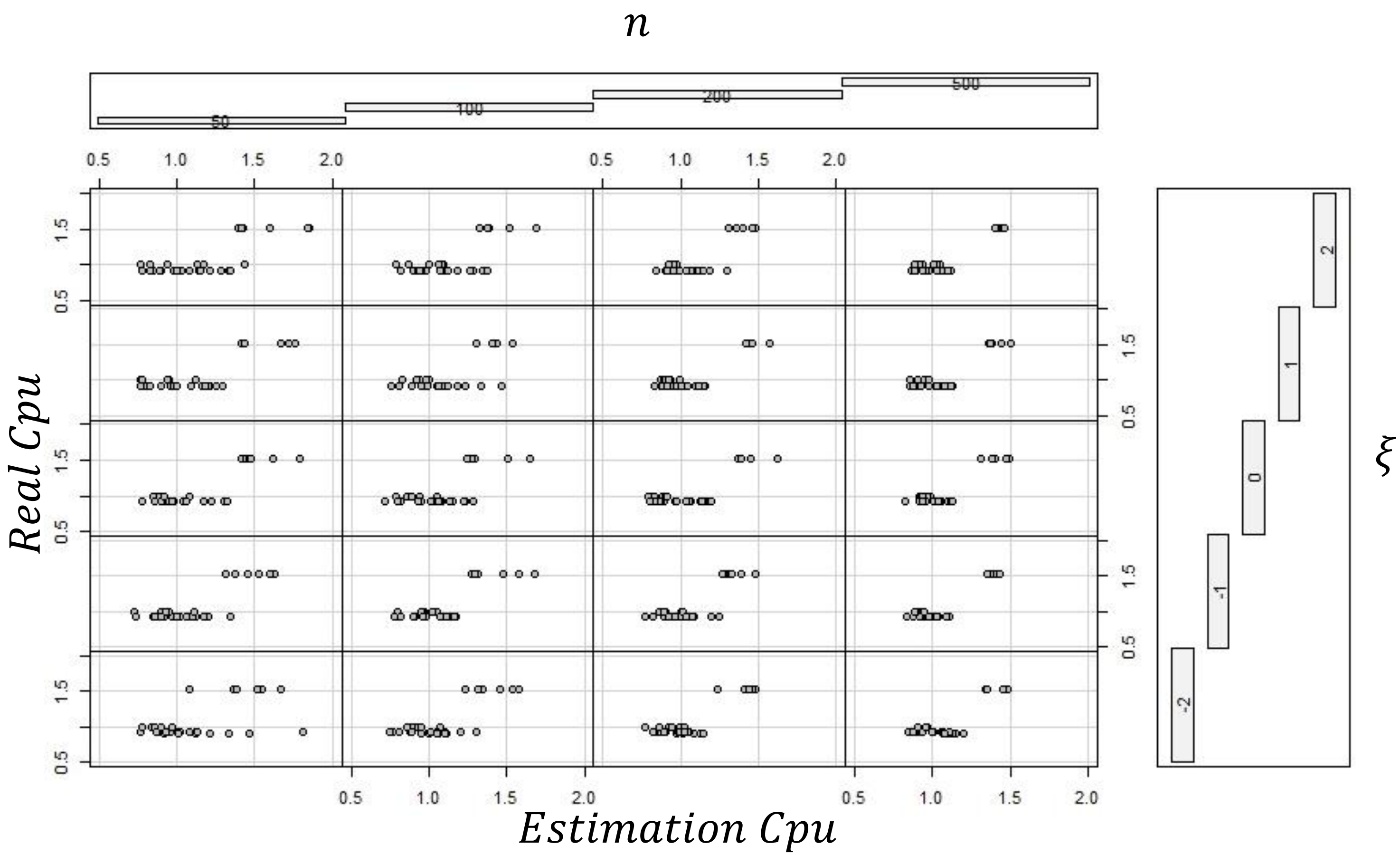


Objectives fulfillment

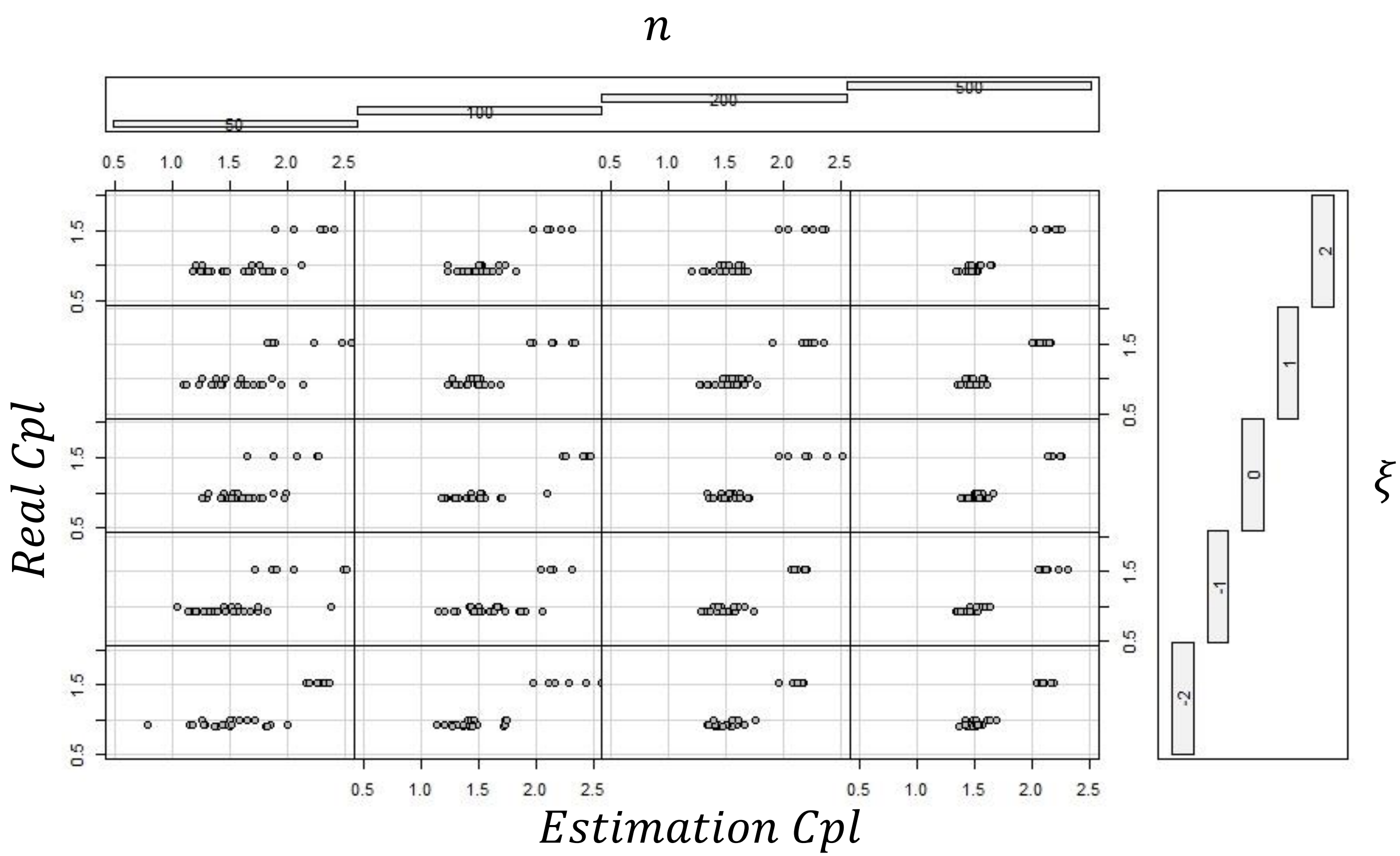
Objectives	Percentage
Identify methods to estimate process capability indices associated with non-normal data.	100%
Select one of these methods and adapt it to the Skewed Normal distribution.	100%
Develop the proposed methodology in a programming language.	80%
Compare the proposed methodology performance against conventionally used methods reported in literature.	10%

THANKS FOR YOUR ATTENTION

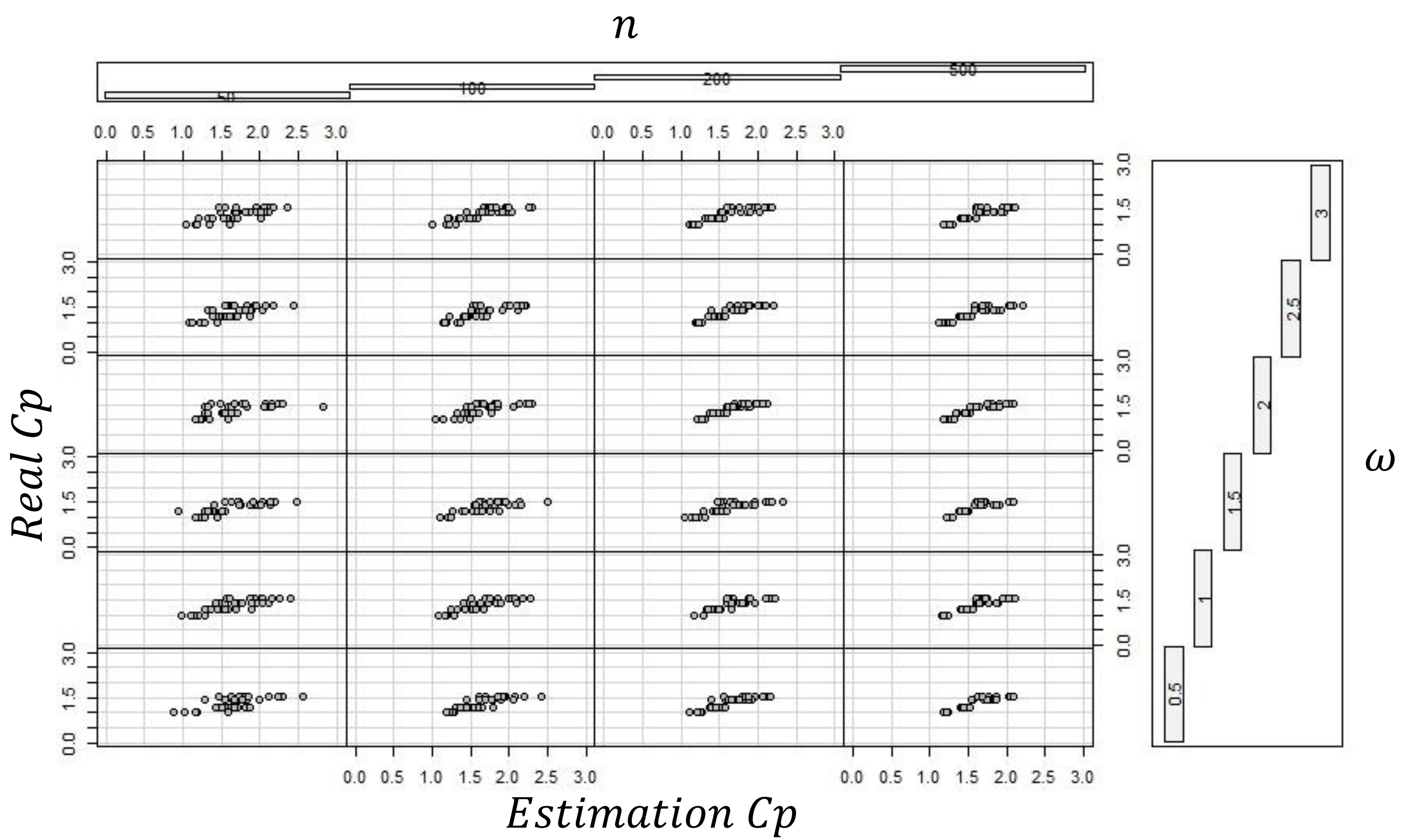
Consistent estimators are obtained for any value of ξ



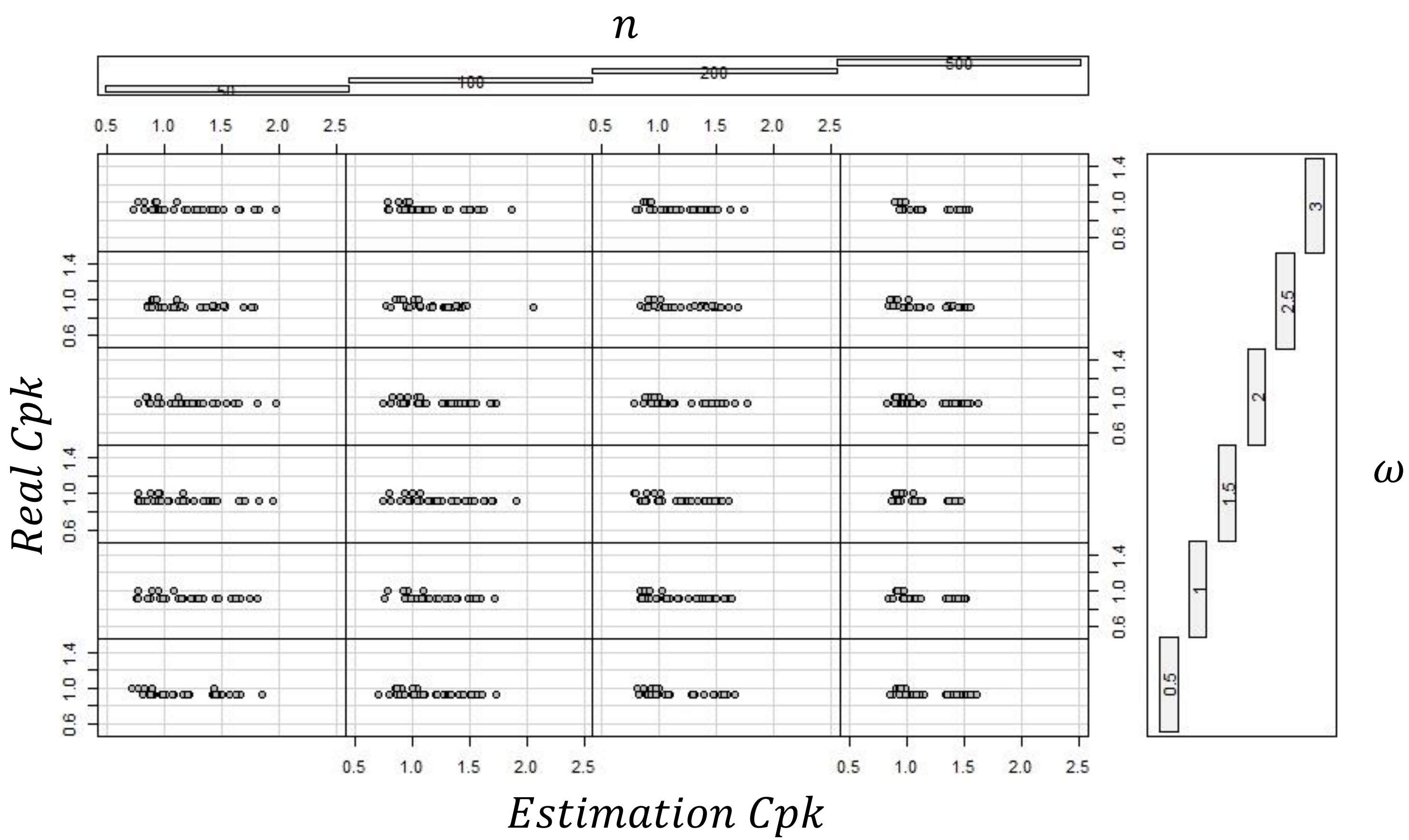
Consistent estimators are obtained for any value of ξ



Consistent estimators are obtained for any value of ω



Consistent estimators are obtained for any value of ω



The snormFit function produces inaccurate estimates of α