Black-Litterman model: Colombian stock market application

Miguel Tamayo-Jaramillo[1]
Susana Luna-Ramírez[2]
Tutor: Diego Alonso Agudelo-Rueda

Research Practise Progress Presentation

EAFIT University, Medellín
Colombia

October 2, 2015

[1] Research Practise 1
[2] Research Practise 2

Black-Litterman (BL) Model

The BL model [Black and Litterman, 1991, 1992] is motivated by the practical failures of the mean-variance optimization model [Markowitz, 1952] where,

- Optimal portfolios weights are unrealistic for implementation.
- The model is very sensitive to the input parameters.

The aim of the Black-Litterman (BL) model is to assign some specific assets and its weights in a portfolio according to different views.

Our Goal

Adapt the BL model to the Colombian stock market to create an optimal portfolio and compare its results to the COLCAP Index.

- Understand the theory behind the model and the computational implementation.
- Define a methodology to incorporate the investors perspectives.
- Build optimal portfolios according to the BL model.
- Compare the historical returns from the market index with the returns from the weights given by the model.

Mathematical Model

The final equation is:

$$w = \hat{\Pi}(\delta\Sigma_p)^{-1}$$

where,

- w is the weights of the portfolio.
- $\hat{\Pi}$ is the estimated expected returns.
- δ is the risk aversion.
- Σ_p is the estimated matrix of covariances of the assets.

Mathematical Model

Prior distribution

$$P(A) \sim N(\Pi, \tau\Sigma) \tag{1}$$

Conditional distribution

$$P(B \mid A) \sim N(P^{-1}Q, P^T\Omega P) \tag{2}$$

where,

- Π is the historical matrix of covariances of the assets.
- τ is the uncertainty of the estimated mean.
- Σ is the matrix of covariances of the assets.
- P is the analysts views matrix.
- Q is the expected returns of the views.
- Ω is the diagonalized matrix of covariances of the views.

Bayes Theorem for the Estimation Model

Given (1) and (2), Bayes Theorem is applied to derive the formula for the posterior distribution of asset returns

$$P(A \mid B) \sim N([(\tau\Sigma)^{-1}\Pi + P^T\Omega^{-1}Q][(\tau\Sigma)^{-1} + P^T\Omega^{-1}Q]^{-1}, \\ [(\tau\Sigma)^{-1} + P^T\Omega^{-1}P]^{-1})$$

$$\hat{\Pi} = \Pi + \tau\Sigma P^T[(P\tau\Sigma P^T) + \Omega]^{-1}(Q - P\Pi)$$
$$M = [(\tau\Sigma)^{-1} + P^T\Omega^{-1}P]^{-1} \tag{3}$$

Computing posterior covariance of returns requires adding the variance about the mean [He and Litterman, 2002], from (3)

$$\Sigma_p = \Sigma + M$$

Calibrating Parameters

Uncertainty$\rightarrow \tau =$ maximum likelihood estimator [Walters, 2014]

$$\tau = \frac{1}{n}$$

Risk aversion $\rightarrow \delta =$ Sharpe ratio [Grinold and Kahn, 2000]

$$\delta = \frac{E[R]_m - R_f}{\sigma_m^2}$$

where,

- n is the number of the data.
- $E[R]_m$ is the expected returns of the market.
- R_f is the risk free rate.
- σ_m^2 is the variance of the market assets.

Data

Time framework: January 2008 to June 2015.

The data taken from Bloomberg is:

- Returns for the assets and the index market.
- Experts perspectives of the assets.
- Weights in the market portfolio.

All of them are in a monthly basis.

Quantifing the Perspectives

Suppose there is an X market index composed by 3 assets. An investor expects that Asset1 outperforms Asset3 by 5%. P, the

perspective vector and Q the vector that includes the relative performance, would be,

$$P = [1 \quad 0 \quad -1]$$
$$Q = 5\%$$

Most of the research papers studied as

[Drobetz, 2001, Black and Litterman, 1992, He and Litterman, 2002] show the BL model performance like the example above, but in practice the perspectives are different, generally, they are numbers that are specified depending if the asset should be bought, sold or hold.

Compute P and Q

[He et al., 2013] suggests to split the perspectives according to the consensus recommendation.

$$\begin{aligned} x < 3 \quad & \textit{Sell} \\ 3 \leq x < 4 \quad & \textit{Hold} - \textit{Buy} \\ 4 \leq x \leq 5 \quad & \textit{Buy} - \textit{Strong buy} \end{aligned}$$

Using this methodology it is possible to build the P matrix with 3 different portfolios, for the Q it would be necessary to compute the difference between the market returns and each of the portfolios returns.

Problem

According to the methodology used in [He et al., 2013], the Colombian stock market analysts do not seem to add value with their perspectives.

Table: Monthly performance average of consensus recommendation. January 2008 - June 2015

Buy - Strong buy Q1	Hold - Buy Q2	Sell Q3
-1.85%	-2.55%	-3.13%

Future Work

- Data snooping for the methodology.
- Define how to evaluate the performance of the model.

Questions?

Work References I


Black, F. and Litterman, R. (1992).
Global portfolio optimization.
Financial Analysts Journal, 48(5):28–43.


Black, F. and Litterman, R. B. (1991).
Asset allocation: combining investor views with market equilibrium.
The Journal of Fixed Income, 1(2):7–18.


Drobetz, W. (2001).
How to avoid the pitfalls in portfolio optimization? Putting the Black-Litterman approach at work.
Financial Markets and Portfolio Management, 15(1):59–75.

Work References II


Grinold, R. C. and Kahn, R. N. (2000).
Active portfolio management: A Quantitative Approach for Producing Superior Returns and Controlling Risk.
McGraw Hill New York, NY.


He, G. and Litterman, R. (2002).
The intuition behind Black-Litterman model portfolios.
Available at SSRN 334304.


He, P. W., Grant, A., and Fabre, J. (2013).
Economic value of analyst recommendations in Australia: an application of the Black-Litterman asset allocation model.
Accounting & Finance, 53(2):441–470.

Work References III


Markowitz, H. (1952).
Portfolio selection.
The journal of finance, 7(1):77–91.


Walters, J. (2014).
The Black-Litterman model in detail.
http://www.blacklitterman.org/Black-Litterman.pdf.
[Online; accessed 26 August, 2015].