Black-Litterman Model: Colombian Stock Market Application

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Research Practise Final Presentation

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November 23, 2015

[1] Research Practise 1

[2] Research Practise 2

Black-Litterman (BL) Model

The BL model [Black and Litterman, 1991, 1992] is motivated by the practical failures of the mean-variance optimization model [Markowitz, 1952] where,

- Optimal portfolios weights are unrealistic for implementation.
- The model is very sensitive to the input parameters.

The aim of the Black-Litterman (BL) model is to assign some specific assets and its weights in a portfolio according to different views. The inclusion of these views are the most important contribution of the model.

BL Overview in Colombia

- [Segura, 2009] applies the BL model to the mandatory pensions funds.
- [León and Vela, 2011] applies the model for the strategic asset allocation in the foreign reserves case.
- [Franco-Arbeláez et al., 2011] examines the theoretical properties of the BL model.

There is no evidence of research papers that in practice make a comparison between the BL model and the traditional models, in order to determine an optimal portfolio on the Colombian stock market.

Our Goal

Adapt the BL model to the Colombian stock market to create an optimal portfolio and evaluate its results against the COLCAP Index.

- Understand the theory behind the model and the computational implementation.
- Design a methodology based on public information to incorporate the investors perspectives.
- Build optimal portfolios according to the BL model.
- Evaluate the BL portfolio performance against the COLCAP.

Mathematical Model

The final equation is:

$$w_{BL} = \hat{\Pi}(\delta\Sigma_p)^{-1}$$

where,

- w_{BL} is the estimated weights from the BL portfolio.
- $\hat{\Pi}$ is the estimated expected returns.
- δ is the risk aversion.
- Σ_p is the estimated matrix of covariances of the assets.

Mathematical Model

Prior distribution

$$P(A) \sim N(\Pi, \tau\Sigma) \tag{1}$$

Conditional distribution

$$P(B \mid A) \sim N(P^{-1}Q, P^{T}\Omega P) \tag{2}$$

where,

- Π is the historical vector of returns of the assets.
- τ is the uncertainty of the estimated mean.
- Σ is the matrix of covariances of the assets.
- P is the analysts views matrix.
- Q is the expected returns of the views.
- Ω is the diagonalized matrix of covariances of the views.

Bayes Theorem for the Estimation Model

Given (1) and (2), Bayes Theorem is applied to derive the formula for the posterior distribution of asset returns

$$P(A \mid B) \sim N([(\tau\Sigma)^{-1}\Pi + P^T\Omega^{-1}Q][(\tau\Sigma)^{-1} + P^T\Omega^{-1}Q]^{-1},$$
$$[(\tau\Sigma)^{-1} + P^T\Omega^{-1}P]^{-1})$$

$$\begin{aligned} \hat{\Pi} &= \Pi + \tau\Sigma P^T[(P\tau\Sigma P^T) + \Omega]^{-1}(Q - P\Pi) \\ M &= [(\tau\Sigma)^{-1} + P^T\Omega^{-1}P]^{-1} \end{aligned} \tag{3}$$

Computing posterior covariance of returns requires adding the variance about the mean [He and Litterman, 2002], from (3)

$$\Sigma_p = \Sigma + M$$

Calibrating Parameters

Uncertainty$\rightarrow$ $\tau=$ maximum likelihood estimator [Walters, 2014]

$$\tau = \frac{1}{n} \simeq \frac{1}{52}$$

- n is the number of the data used for the covariance matrix.

Calibrating Parameters

Risk aversion $\rightarrow$ [Grinold and Kahn, 2000] suggests to use the Sharpe ratio to calculate δ

$$\delta = \frac{Sharpe\ Ratio}{\sigma_{market}} = \frac{E[R]_{market} - R_f}{\sigma^2_{market}} = \frac{Risk\ Premium}{\sigma^2_{market}}$$

- $E[R]_m$ is the expected returns of the market.
- R_f is the risk free rate.
- σ^2_m is the variance of the market assets.

We took the *Risk Premium* from [Damodaran, 2015] $\rightarrow$ highly accepted on the financial environment.

Data

Time framework: January 2009 to June 2015.

The data taken from Bloomberg is:

- Daily last price for all the assets (*PX_LAST*).
- Experts perspectives of the assets (*EQY_REC_CONS*).
- Weights in the market portfolio.
- One year average objective price (*BEST_TARGET_PRICE*).

The last three items are in a monthly basis.

- Colombian Risk Premium anual rate from Damodaran web page.
- Risk free anual rate taken from the bank of the Republic of Colombia web page.

Quantifing the Perspectives

Suppose there is an X market index composed by 3 assets. An investor expects that Asset1 outperforms Asset3 by 5%. P, the perspective vector and Q the vector that includes the relative performance, would be,

$$P = \begin{bmatrix} 1 & 0 & -1 \end{bmatrix}$$
$$Q = 5\%$$

Most of the research papers studied as [Drobetz, 2001, Black and Litterman, 1992, He and Litterman, 2002] show the BL model performance like the example above, but in practice the perspectives are different, generally, they are numbers that are specified depending if the asset should be bought, sold or hold.

Compute P and Q

[He et al., 2013] suggests to split the perspectives according to the consensus recommendation.

$$
\begin{array}{rl}
x < 3 & \quad \textit{Sell} \\
3 \leq x < 4 & \quad \textit{Hold} - \textit{Buy} \\
4 \leq x \leq 5 & \quad \textit{Buy} - \textit{Strong buy}
\end{array}
$$

Using this methodology it is possible to build the P matrix with 3 different portfolios, for the Q matrix it would be necessary to compute the difference between the market returns and each of the portfolios returns.

P & Q According to Our Methodology

Suppose that a market index is composed by 7 assets, the 7 assets have the same weights in the portfolio. The average of the analysts think assets 1, 2, 3 should be bought, 4 and 5 should be hold and finally 6 and 7 should be sold. The P matrix would be,

$$P = \begin{bmatrix} 33.3\% & 33.3\% & 33.3\% & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 50\% & 50\% & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 50\% & 50\% \end{bmatrix}$$

Also they think that in average the assets from the first group in x years will have a return of 15%, from the second group of 2% and the third one of -10%. The Q matrix would be,

$$Q = \begin{bmatrix} 15\% & 2\% - 10\% \end{bmatrix}$$

Problem

According to the methodology used in [He et al., 2013], the Colombian stock market analysts do not seem to add value with their perspectives. So we use the objective average prices to compute the Q matrix.

Table: Monthly performance average of consensus recommendation. Using objective average prices. January 2009 - June 2015

	Sell Q1	**Hold - Buy Q2**	**Buy - Strong buy Q3**
[He et al., 2013]	-3.13%	-2.55%	-1.85%
Objective prices	-0.56%	3.11%	3.60%

BL Evaluation

Table: Quarterly returns. Perspectives portfolios vs. COLCAP. January 2009 - January 2015

	R[P_{sell}]	R[P_{hold}]	R[P_{buy}]	COLCAP
Average	-1.03%	1.32%	2.69%	1.37%
α	-1.78%	0.08%	2.373%	
R^2	57.39%	70.05%	33.11%	

Table: Quarterly returns. Black-Litterman estimate Portfolio vs COLCAP. January 2009 - April 2015

	R[BL]
Average	2.80%
α	1.69%
R^2	88.70%
p-value	0.0198

Properties Evaluation

To check if BL is robust we perform a sensibility analysis in Risk aversion parameter (δ).

Table: Sensibility analysis

	$+5\%$	$+10\%$	-5%	-10%
α	1.68%	1.68%	1.69%	1.69%
p-value	0.0188	0.0179	0.0208	0.0218

BL estimated portfolios are also intuitive.

Conclusions

- According to the methodology proposed in average the BL quarterly returns beats the market index (COLCAP) and it has a positive alpha, statistically different from zero.
- In practice, the BL model is robust and intuitive.
- The average consensus recommendations are a good proxy to determine the analyst perspectives, it was proved that in Colombia this recommendations are consistent. Also, the average objective prices established by the analyst are a good methodology to estimate the expected returns (Q matrix).

Future Work

- Other sensibility exercises, this means estimates other optimal portfolios as the Sharpe Ratio and the minimum risk portfolio.
- Estimate the covariances matrix with different methodologies.

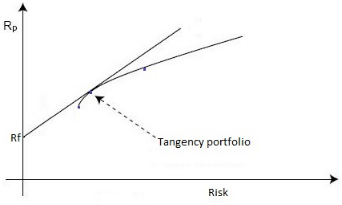

Figure: Efficient frontier

Questions?

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