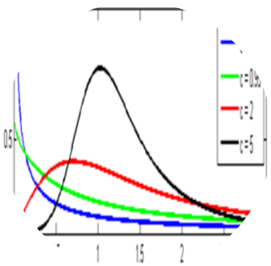
ANALYSIS OF PROCESSES CAPABILITY USING THE SKEWED NORMAL DISTRIBUTION

Susana Agudelo Jaramillo

Myladis Rocio Cogollo Flórez
Juan Miguel Cogollo Flórez

Research Practise 1
EAFIT University
November 23th, 2015

Process capability indices requires normality



In practice there are interest variables that do not follow a normal distribution.

$$C_p = \frac{USL - LSL}{6\,\sigma}$$

Traditional process capability indices are sensitive to non-normality of data.

Methods to estimate process capability indices (PCI) associated with non-normal data

Clements's percentile method:

It calculates the indices using a family of Pearson curves [1].

Box-Cox transformation method:

It consists of an initial data transformation followed by the application of conventional methods to resulting data considered as normal [2].

[1] Clements, J. A. (1989). "Process capability indices for non-normal calculations". Quality Progress, 22, 49-55.
[2] Ahmad, S., Abdollahian, M. and Zeephongsekul, P. (2008). "Process capability estimation for non – normal quality characteristics: A comparison of Clements, Burr and Box – Cox Methods". ANZIAM Journal, 49, 642–665.

Clements's percentile method

Index [1]	Normal	Non-Normal
Potential process capability index.	$C_p = \frac{USL - LSL}{6\sigma}$	$C_p = \frac{USL - LSL}{P_{0.99865} - P_{0.00135}}$
Capability index for the lower specification level.	$C_{pl} = \frac{\mu - LSL}{3\sigma}$	$C_{pl} = \frac{P_{0.5} - LSL}{P_{0.5} - P_{0.00135}}$
Capability index for the upper specification level.	$C_{pu} = \frac{USL - \mu}{3\sigma}$	$C_{pu} = \frac{USL - P_{0.5}}{P_{0.99865} - P_{0.5}}$
Real process capability index.	$C_{pk} = min\{C_{pl}, C_{pu}\}$	$C_{pk} = min\{C_{pl}, C_{pu}\}$

Where:

- μ: mean
- σ^2: variance
- USL and LSL: upper and lower specification limits
- P_q: q percentile, with $0 < q < 1$.

[1] Clements, J. A. (1989). "Process capability indices for non-normal calculations". Quality Progress, 22, 49-55.

Skewed Normal distribution

The probability density function associated to a random variable with a Skewed Normal distribution is as follows [3]:

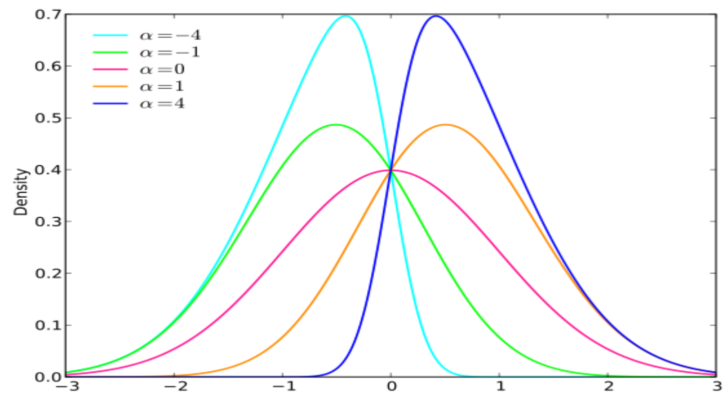
$$f(x) = \frac{1}{\omega\pi} e^{-\frac{(x-\xi)^2}{2\omega^2}} \int_{-\infty}^{\alpha\left(\frac{x-\xi}{\omega}\right)} e^{-\frac{t^2}{2}} \, dt$$

Where:

ξ is a position parameter ⟷ μ

ω is a scaling parameter ⟷ σ

α is a shape parameter.



Taken from: www.wikipedia.com

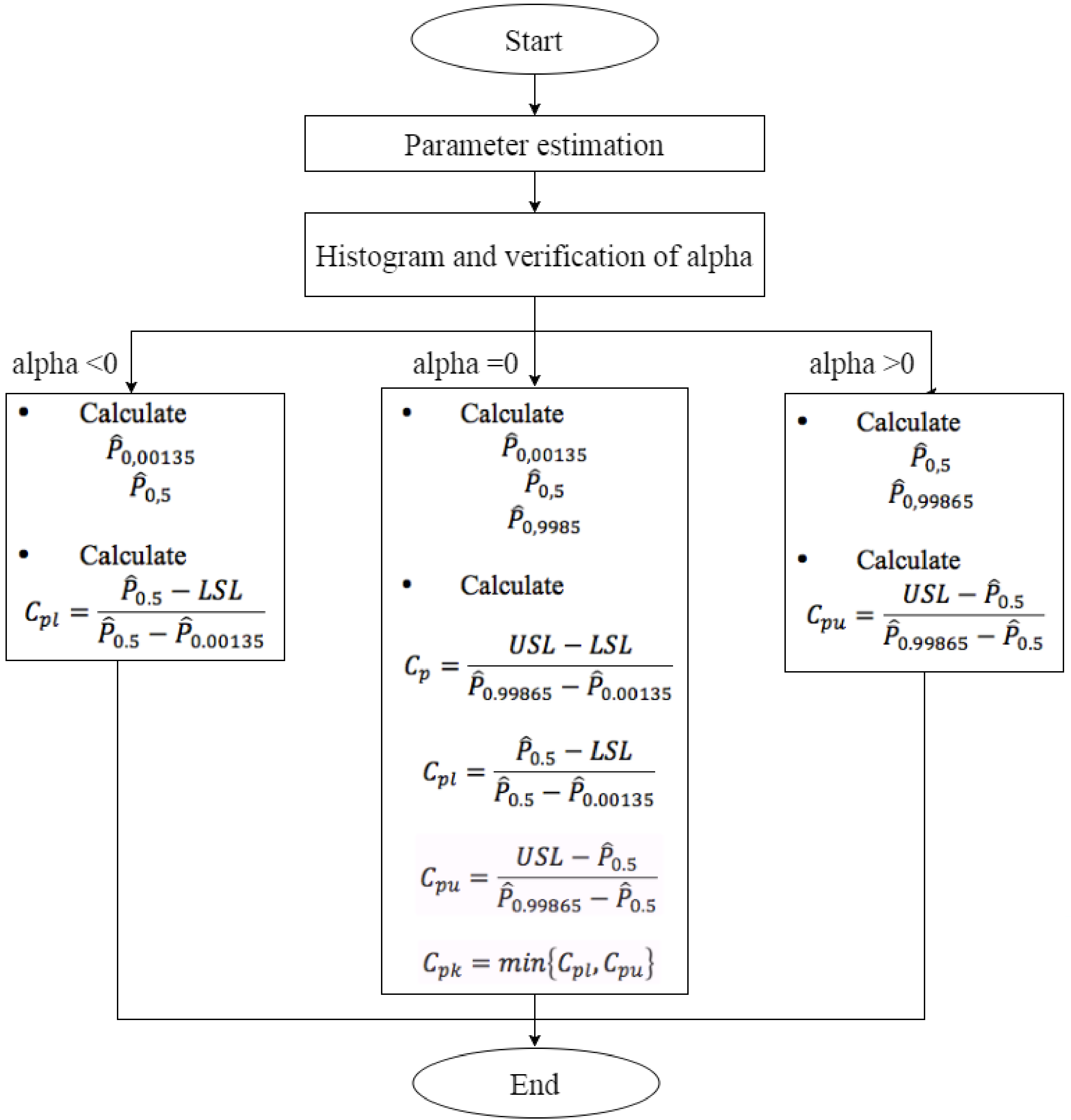
$\alpha < 0$ → Negative asymmetry.

$\alpha = 0$ → Symmetry.

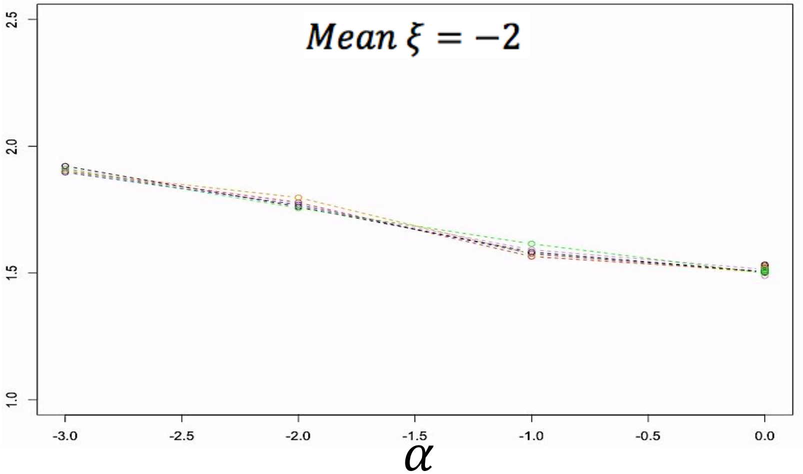
$\alpha > 0$ → Positive asymmetry.

[3] Figueiredo, F. and Gomes, I. (2011) "The skew-normal distribution in SPC". National Funds through Fundação para a Ciência e a Tecnologia.

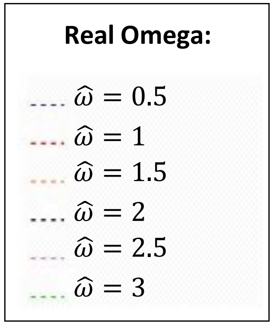
PCI calculation method using Skewed Normal distribution

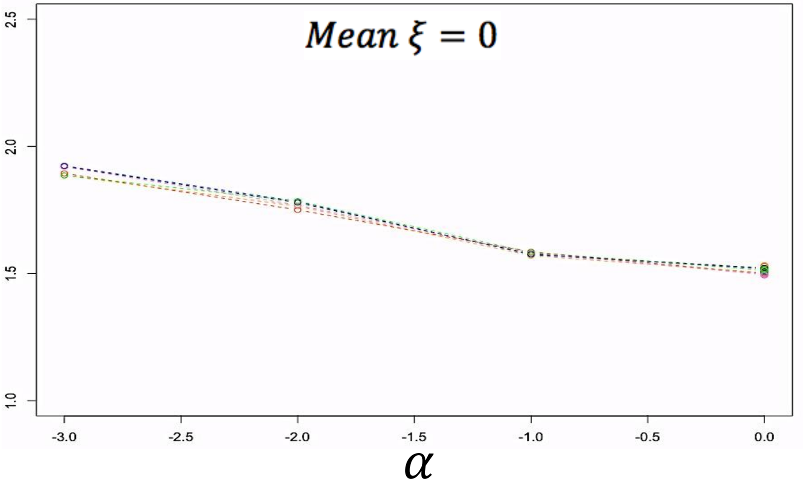


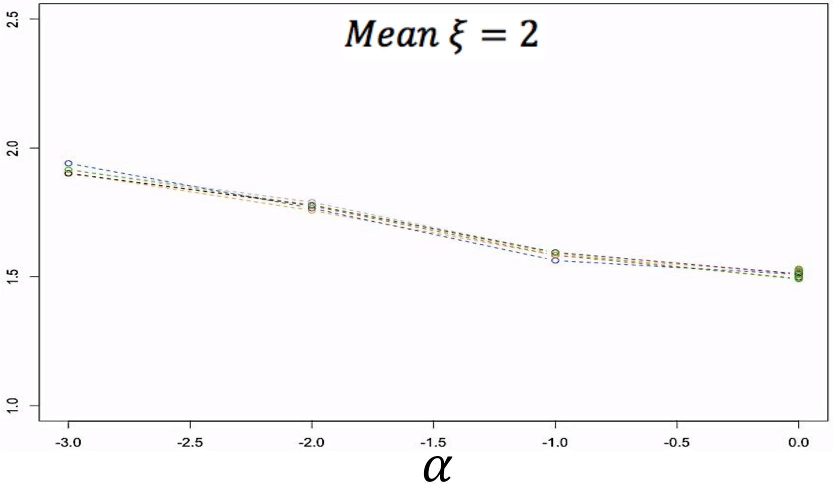
The shape parameter (α) sets a trend in estimates of process capability indices



C_{pl} *index for sample size* $n = 500$

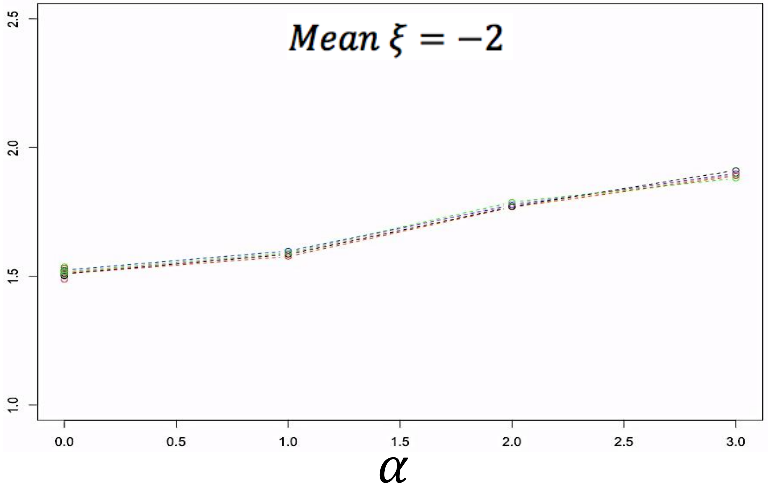






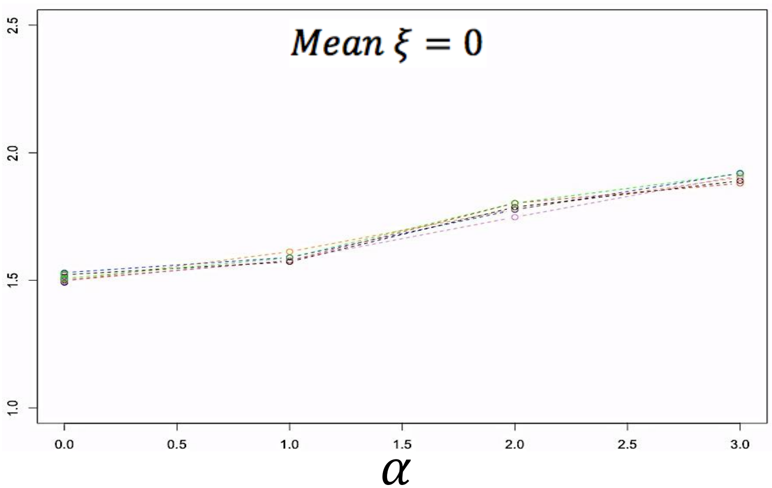
The shape parameter (α) sets a trend in estimates of process capability indices

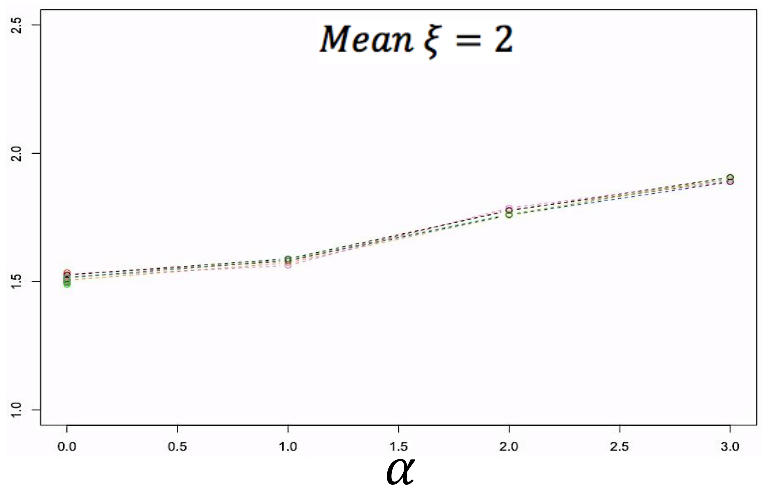
C_{pu} *index for sample size* $n = 500$



Real Omega:

- $\widehat{\omega} = 0.5$
- $\widehat{\omega} = 1$
- $\widehat{\omega} = 1.5$
- $\widehat{\omega} = 2$
- $\widehat{\omega} = 2.5$
- $\widehat{\omega} = 3$





Simulation study

The simulation was as follows:

1. Set values of:

$$C_{pl}, C_{pu} = \{0.5,1,1.5,2\}$$

2. Set parameters:

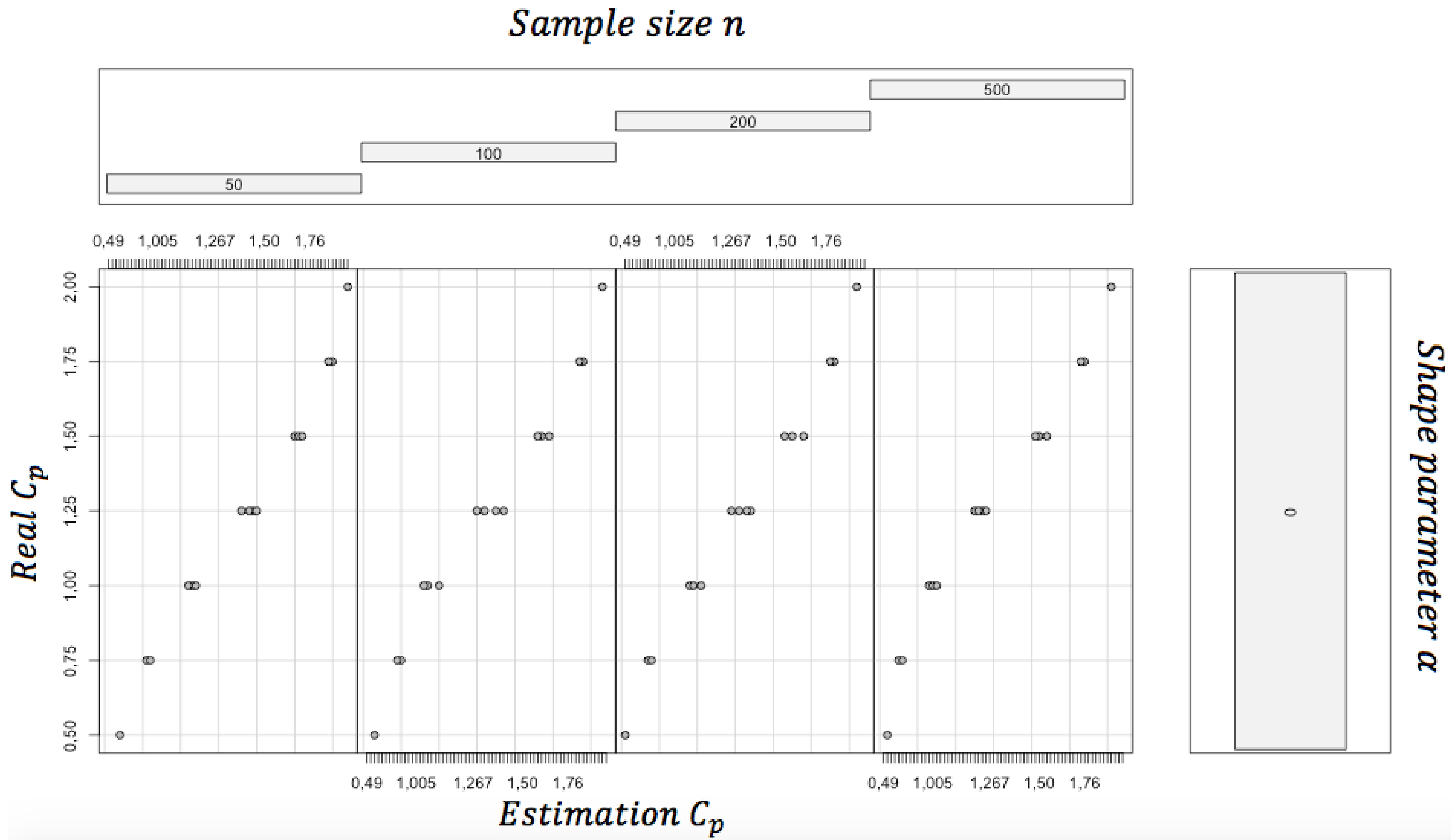
$$\xi = 2$$
$$\omega = 0.5$$
$$\alpha = \{-3,-2,-1,0,1,2,3\}$$

3. Calculate specification limits:

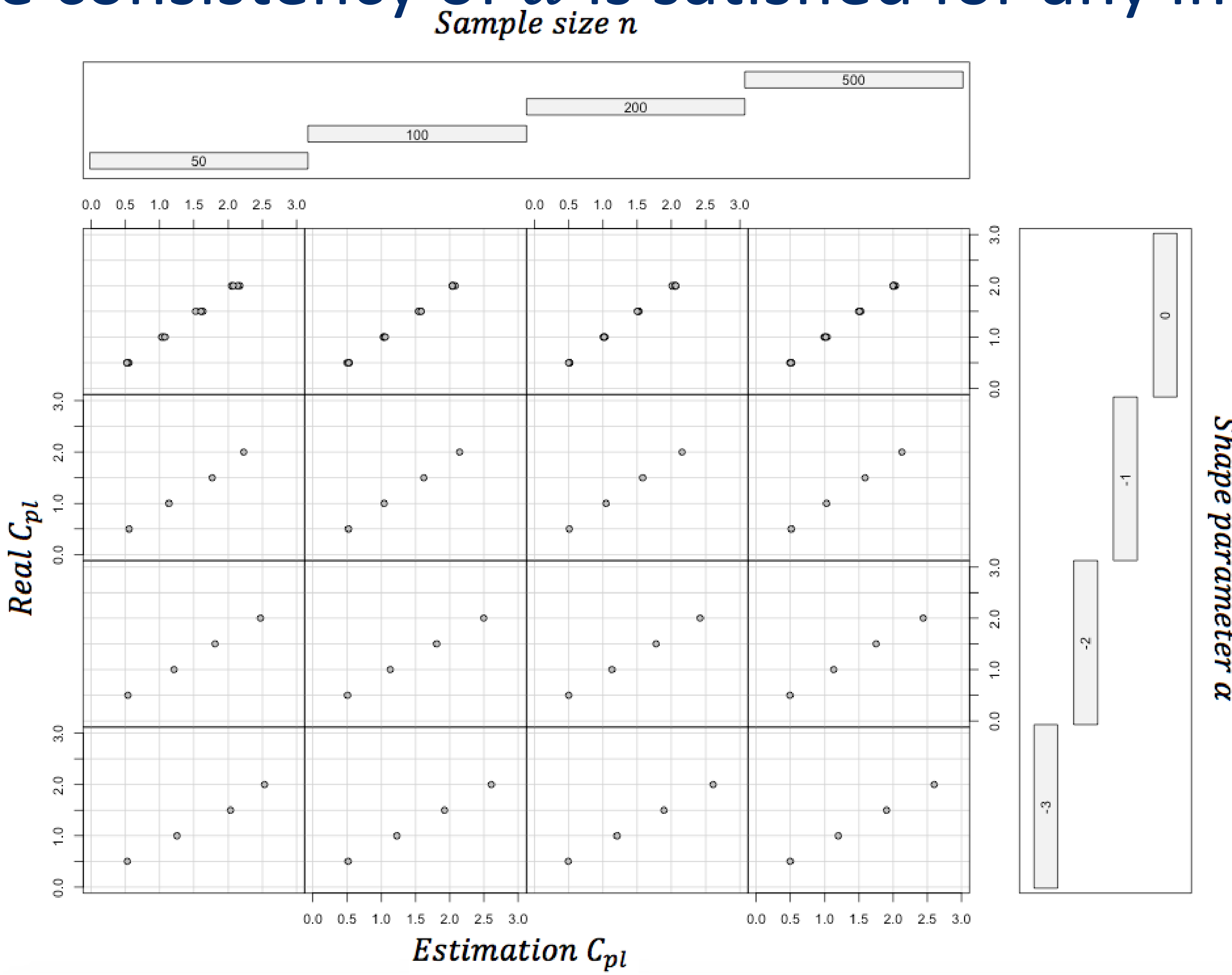
$$LSL = P_{0,5} - C_{pl}(P_{0,5} - P_{0,00135})$$
$$USL = C_{pu}\left(P_{0,99865} - P_{0,5}\right) + P_{0,5}$$

4. Generated skewed normal data with set parameters ξ, ω and α.
5. Estimate PCI as described in Flowchart.

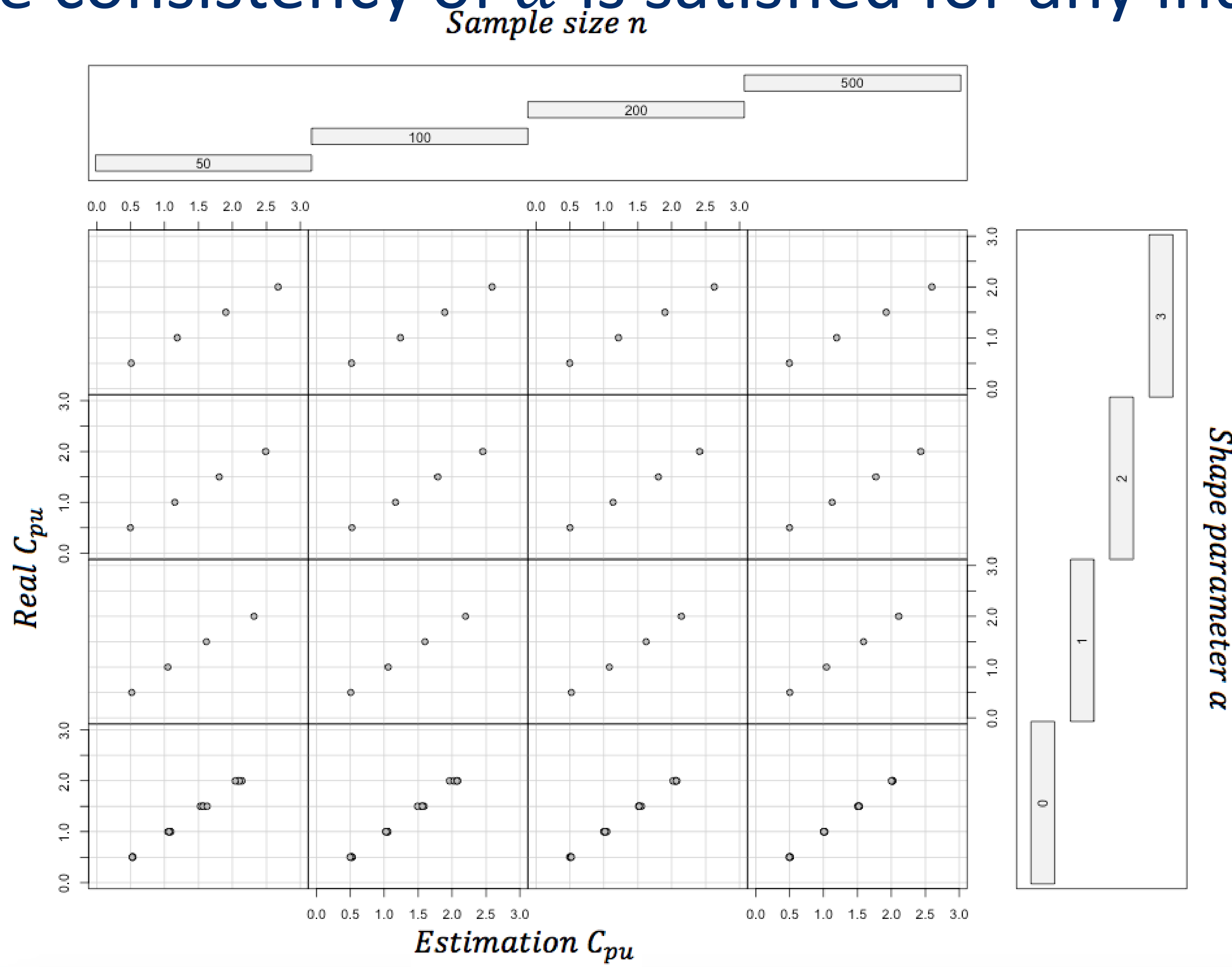
Consistent estimators are obtained for any value of α



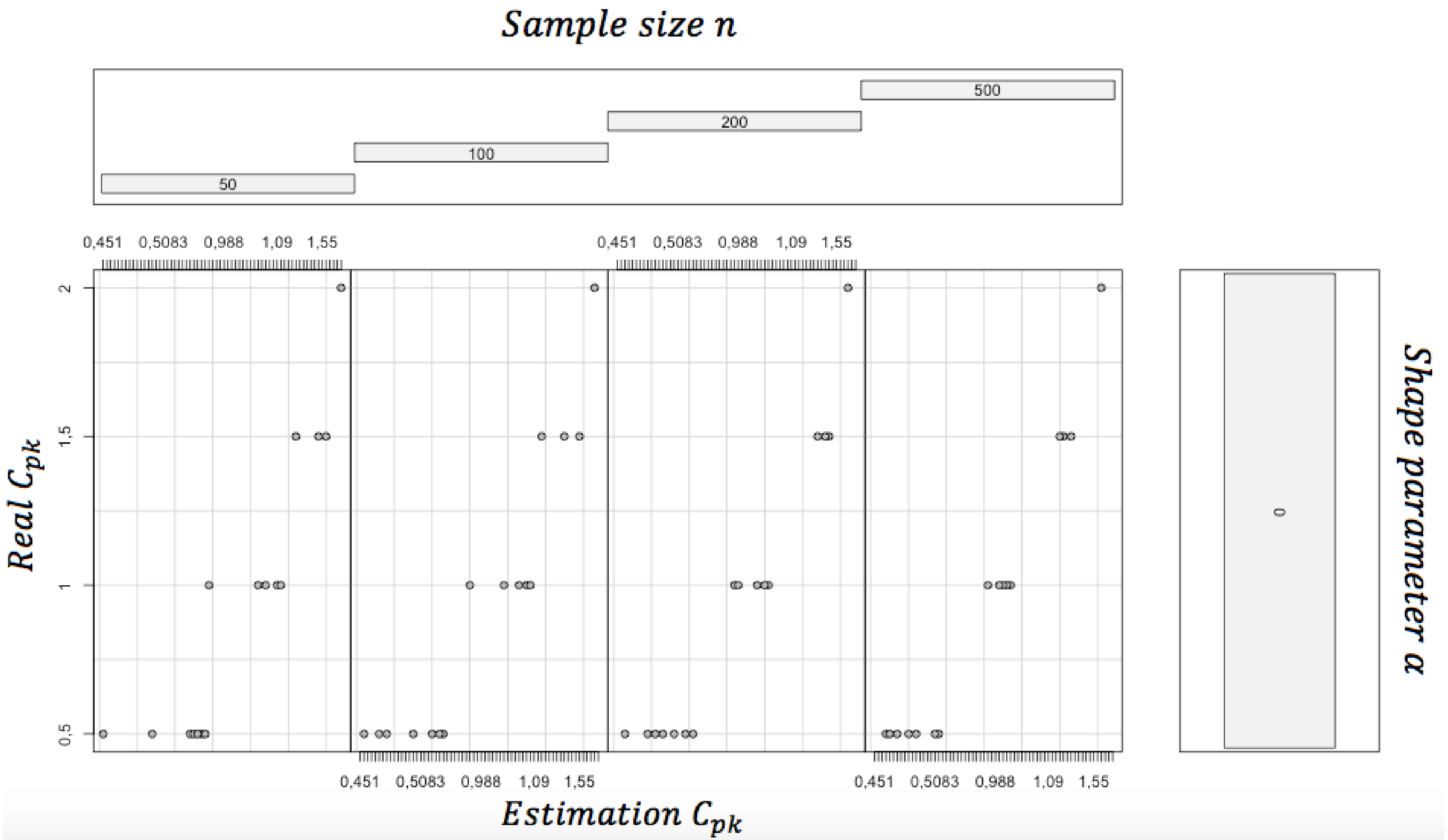
The consistency of α is satisfied for any index



The consistency of α is satisfied for any index



The consistency of α is satisfied for any index



Mean and standard deviation for 30 samples in C_{pl} index

		50		100		200		500	
α	**Real C_{pl}**	***Mean***	***sd***	***Mean***	***sd***	***Mean***	***sd***	***Mean***	***sd***
-3	**0,5**	0,530	0,126	0,513	0,080	0,507	0,055	0,490	0,035
	1	1,302	0,238	1,201	0,125	1,235	0,096	1,201	0,051
	1,5	2,046	0,320	1,917	0,156	1,931	0,118	1,902	0,079
	2	2,606	0,271	2,622	0,225	2,588	0,169	2,623	0,119
-2	**0,5**	0,515	0,121	0,505	0,090	0,505	0,054	0,509	0,034
	1	1,260	0,177	1,147	0,108	1,137	0,089	1,130	0,040
	1,5	1,806	0,273	1,811	0,139	1,798	0,105	1,795	0,077
	2	2,600	0,363	2,399	0,189	2,437	0,140	2,417	0,089
-1	**0,5**	0,524	0,167	0,513	0,069	0,489	0,065	0,489	0,032
	1	1,063	0,154	1,072	0,109	1,054	0,081	1,049	0,050
	1,5	1,720	0,208	1,637	0,143	1,602	0,101	1,603	0,055
	2	2,255	0,210	2,182	0,187	2,134	0,107	2,142	0,078

Mean and standard deviation for 30 samples in C_{pu} index

		50		100		200		500	
α	**Real C_{pu}**	***Mean***	***sd***	***Mean***	***sd***	***Mean***	***sd***	***Mean***	***sd***
1	**0,5**	0,522	0,141	0,497	0,097	0,496	0,060	0,505	0,030
	1	1,083	0,186	1,094	0,098	1,042	0,087	1,042	0,048
	1,5	1,683	0,255	1,580	0,131	1,606	0,100	1,581	0,066
	2	2,283	0,362	2,192	0,206	2,170	0,131	2,122	0,080
2	**0,5**	0,502	0,125	0,533	0,111	0,519	0,078	0,489	0,034
	1	1,146	0,130	1,170	0,106	1,158	0,089	1,148	0,059
	1,5	1,906	0,284	1,794	0,153	1,781	0,126	1,776	0,070
	2	2,415	0,329	2,461	0,204	2,444	0,165	2,416	0,087
3	**0,5**	0,524	0,136	0,501	0,075	0,511	0,064	0,494	0,034
	1	1,209	0,178	1,165	0,112	1,203	0,083	1,193	0,056
	1,5	1,940	0,201	1,933	0,207	1,909	0,109	1,875	0,079
	2	2,712	0,316	2,649	0,218	2,617	0,149	2,590	0,101

THANKS FOR YOUR ATTENTION