IMPLEMENTATION OF MULTI-OBJECTIVE INTERVAL VALUED OPTIMIZATION TECHNIQUES APPLIED TO PARAMETER ESTIMATION UNDER UNCERTAINTY

AUTHOR: JOSE D. GALLEGO-POSADA[1]

PROJECT ADVISOR: PROF. MARIA E. PUERTA-YEPES[2]

MATHEMATICAL ENGINEERING
DEPARTMENT OF MATHEMATICAL SCIENCES
UNIVERSIDAD EAFIT
MEDELLÍN

FEBRUARY 2015

1 *Mathematical Engineering, EAFIT University, Medellín, Colombia*
2 *Department of Mathematical Sciences, EAFIT University, Medellín, Colombia*

CONTENTS

1 PROBLEM DESCRIPTION

Let $I(\mathbb{R}) = \{[a, b] : a \leqslant b,\ a,\ b \in \mathbb{R}\}$ be the set of all closed intevals of $\mathbb{R}$. Note that $I(\mathbb{R})$ is a natural extension of the set of real numbers in which we can identify every real number x with the interval $[x, x]$. We say a function f is interval valued if $f : \mathbb{R} \to I(\mathbb{R})$.

Consider the mathematical model of a system given by the function $g(x, \Theta)$, where $\Theta \in \mathbb{R}^n$ is the vector of parameters of the system. Given experimental data $\{y_i\}$ (inaccurate because the influence of randomness and uncertainty), we would like to estimate the values of those parameters, so that the model attains the optimal fitting to the observed data.

In the usual estimation techniques, a vector of real numbers is obtained (one value for each of the parameters involved), but the probability that the estimation result happens to be the actual value of the parameter is very low and is, therefore, very sensitive to perturbations.

However, the use of interval valued analysis allows us to provide regions in which, given a determined confidence level, we can ensure the enclosing of the solution, thus reducing the impact of random disturbances and computational errors in the final result. In this case, the result of the estimation is an interval in $I(\mathbb{R})$, instead of a single element of $\mathbb{R}$.

In addition, it is possible to analyze optimization problems in which the uncertainty is included in the formulation as intervals in the coefficients of the objective function but also some cases in the constraints of the problem, as well as a generalization of the problem to the multi-objective optimization context, i.e.

$$\min\ f_i(x) \qquad \forall\, i \in U$$

subject to,

$$h_i(x) \preceq 0 \qquad \forall\, i \in V$$

$$k_i(x) = [0, 0] \qquad \forall\, i \in W$$

$$x \in D \subset \mathbb{R}^n$$

where f_i, h_i, $k_i : \mathbb{R}^n \to I(\mathbb{R})$, U, V and W are indexing sets, D is the domain set of the independent variables and $\preceq$ is an order relation.

2 OBJECTIVES

2.1 General Objectives

Perform a computational implementation of multi-objective interval valued techniques applied to a problem of parameter estimation under uncertainty.

2.2 Specific Objectives

1. Elaborate a report that describes the main techniques employed in multi-objective optimization and interval valued optimization.
2. Pose a problem requiring parameter estimation under uncertainty.
3. Perform a computational implementation of the multi-objective interval valued optimization techniques applied to the mentioned estimation problem.
4. Redact a final report detailing the results of the present work.

3 JUSTIFICATION

The vast majority of mathematical models developed to represent various problems depend on sets of parameters whose values are generally determined based on experimental measurements of them, or inferred according to datasets obtained through observation of the phenomenon.

Thus, the uncertainty in those values due to measurement errors, rounding in calculations, computational representability and even possibly because of a lack of knowledge of qualitative information associated with those parameters, makes the interval analysis and, particularly, the interval-valued optimization a very useful tool in the development of efficient techniques that solve this kind of problems and reduce undesired effects as the sensitivity of the obtained results based on those analyzes.

On the other hand, a large number of applications of optimization in real-life problems encompasses situations whose characteristics require the simultaneous pursuing of potentially conflicting objectives. Clearly, this increases the level of detail within the mathematical model by augmenting the complexity of it and, possibly, the difficulty in the search of efficient solutions.

A mixture of both tools could lead to the establishment of more robust techniques when facing the mentioned difficulties, obtaining more reliable and rigorous results in a mathematical perspective.

4 PRECEDING RESEARCH

The foundations of interval analysis were established by Ramon Moore [1] in his PhD Disertation in 1963. Based on that work and the advances provided by Skelboe [2], Hansen [3] and Stroem [4], among others, several authors

(Ratschek & Voller [5], Bhurjee & Panda [6]) have studied the potential of interval valued techniques in the field of optimization. On the other hand, authors like Ichida & Fujii [7] or Karmakar [8] have developed significant research related to the implementation of multiobjective techniques including interval valued optimization. However, there is a lack of efforts in the implementation of these procedures in the context of parameter estimation which is one of the principal aims of this research work.

5 SCOPE

This project is a first approach to the multi-objective interval valued optimization techniques, in order to develop a generalization of the interval optimization methods towards a representation in terms of parametrizations (a natural parametrization of an interval is $[a, b] = f(t) = a + (b - a)t$ with the parameter t varying in the range $[0, 1]$). However, this goal may require further mathematical background as well as a longer time span that would not be adequate to embbed in a single semester research. Therefore, the possibility to continue with the generalization mentioned in another research practice remains open.

6 METHODOLOGY

In order to ensure a proper development of the project, there will be 5 hours per week available to be focused in the current research. This time includes weekly meetings between the tutor and the practitioner to assess progress regarding the research and mathematical development required, review and discussion of literature in the first phase and subsequent amendments to the model and computational implementations in software packages as well as assignment of additional tasks related to the research practice.

7 RESOURCES

Due to the nature of the project, a high percentage of the activities are focused in theoretical mathematical developments, however, during the exploratory stages, access to databases provided by the University is required. Similarly, it is important to consider the usage of software packages, licensed to the University, that allow rigorous and efficient interval arithmetic calculations to be performed in the application of the project to a concrete problem.

8 SCHEDULE

Activity	Description	Product	Start	End
RP	Formal definition of the project and initial approaches to the project proposal.	Research project	Week 1	Week 2
RP	Written and verbal presentation of the research proposal	Project proposal. - Presentation	Week 3	Week 4
Objective 1	Study of the mathematical structure of interval analysis and multi-objective optimization techniques.	Literature review report	Week 5	Week 8
Objective 2	Selection of a problem requiring parameter estimation under uncertainty	Application context	Week 8	Week 10
RP	Oral progress report	Presentation	Week 10	Week 10
Objetive 3	Computational implementation of the mentioned techniques in the selected problem.	Software	Week 9	Week 14
Objective 4	Writing and review of the final report of the project including the results achieved	Final article	Week 1	Week 17
RP	Presentation regarding the results of the project developed during the semester.	Project presentation	Week 19	Week 19

9 INTELLECTUAL PROPERTY

According to the internal regulations on intellectual property within Universidad EAFIT, the results of this practise are product of the co-autorship between Prof. Dr. María Eugenia Puerta Yepes and student Jose Daniel Gallego Posada.

In case further products, beside academic articles, should be generated from this work, the intellectual property distribution related to them will be directed under the current regulation of this matter determined by Universidad EAFIT [9].

10 REFERENCES

[1] R. E. Moore, *Interval Arithmetic and Automatic Error Analysis in Digital Computing*. PhD thesis, Stanford, CA, USA, 1963. AAI6304614.

[2] S. Skelboe, "Computation of rational interval functions," *BIT*, vol. 14, no. 1, pp. 87–95, 1974.

[3] E. Hansen, "Global optimization using interval analysis ? the multi-dimensional case," *Numer. Math.*, vol. 34, no. 3, pp. 247–270, 1980.

[4] T. Ström, "Strict estimation of the maximum of a function of one variable," *BIT*, vol. 11, no. 2, pp. 199–211, 1971.

[5] H. Ratschek and R. Voller, "What can interval analysis do for global optimization?," *Journal of Global Optimization*, vol. 1, no. 2, pp. 111–130, 1991.

[6] A. Bhurjee and G. Panda, "Efficient solution of interval optimization problem," *Mathematical Methods of Operations Research*, vol. 76, no. 3, pp. 273–288, 2012.

[7] K. Ichida and Y. Fujii, "An interval arithmetic method for global optimization," *Computing*, vol. 23, no. 1, pp. 85–97, 1979.

[8] S. Karmakar and A. K. Bhunia, "An alternative optimization technique for interval objective constrained optimization problems via multiobjective programming," *Journal of the Egyptian Mathematical Society*, vol. 22, no. 2, pp. 292–303, 2014.

[9] Universidad EAFIT, "Reglamento de Propiedad Intelectual," 2009.