NURSE SCHEDULING PROBLEM

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AGENDA

1. Introduction
2. State of the art
3. Nurse Scheduling Competition
4. Problem Definition
5. Mathematical Model
6. Conclusions and future work

INTRODUCTION

- Very important when managing all kinds of employees and resources.
- Specially important and complex to healthcare professionals.
- As seen in Burke et al.(2004), scheduling approached by several investigators for more than 40 years.
- Until recently solved manually in a very time consuming process.
- First papers were based on a strictly mathematical approach.
- Heuristic and metaheuristic approaches are seen later.

STATE OF THE ART

- As observed in (Burke et al. 2004) many different approaches have been used to solve the NSP
  - Stochastic programming
  - Linear and quadratic models
  - ANSOS
  - Multi objective
  - Expert systems and artificial intelligence
  - All kinds of heuristics

NURSE ROSTERING COMPETITION

INRC-II

The Second International Nurse Rostering Competition

- International competition
- Second iteration
- Combinatorial Optimization and Decision Support (CODeS)
  - Kulak University (Belgium)
  - Patrick De Causmaecker
- Scheduling and timetabling group
  - University of Uldine (Italy)
  - Sara Cheschia and Andrea Schaerf
- Methodologies for Optimization and Decision Support in the Healthcare Sector (MoBiZ)
  - Vives University (Belgium)
  - Stefaan Haspeslagh

TOP OF LAST COMPETITION

Valois et al. (2010)	• Strictly mathematical approach • Partition into 2 sub problems or phases
Nonobe (2010)	• Metaheuristic for a COP • Tabu search with an easy transformation • Use of binary variables
Zhipeng and Jin-Hao (2010)	• Adaptive local search • Multi start • Diferent neighborhoods • Different strategies to explore neigborhoods
Burke et al. (2010)	• Use of previously developed staff rostering model • Variable depth search and branch and pricing

PROBLEM DEFINITION

- According to Chesia et al.(2014)
- Basic problem
  - Weekly scheduling of a fixed number of nurses.
  - Each day split in shifts.
  - Skills with different requirements.

  General Problem
  - Solution to the problem for a set of n weeks.
  - Requests of the nurses accounted for as soft constraints.
  - History of every week and overall history to account for contractual constraints.

MATHEMATICAL MODEL

$$\min Z = \Delta Z_1 + \Delta Z_2 + \Delta Z_3 + \Delta Z_4 + \Delta Z_5 + \Delta Z_6 + \Delta Z_7 \quad (1)$$

$$s.t. \sum_{s \in S} \sum_{k \in K} x_{nsdk} = 1, \quad \forall\ n \in N,\ d \in D \quad (2)$$

$$\sum_{n \in N} x_{nsdk} \cdot r_{nk} \geq RM_{sdk}, \quad \forall\ s \in S,\ d \in D,\ k \in K \quad (3)$$

$$\sum_{k \in K} (x_{n,s_1,d-1,k} + x_{n,s_2,d,k}) \leq 1, \quad \forall\ n \in N,\ d \in D \backslash \{1\},\ (s_1, s_2) \in P \quad (4)$$

Main Model

- Binary variables x_{nsdk} for nurse n in shift s on day d with skill s.
- Hard restrictions (2) to (4).
- Use of soft constraints as decision variables ΔZ_i.
- RM: Required Minimum nurses

MATHEMATICAL MODEL

$$\sum_{n \in N} x_{nsdk} \cdot r_{nk} + M_{sdk} \geq RO_{sdk}, \quad \forall\ s \in S,\ d \in D,\ k \in K \qquad (5)$$

$$\Delta Z_1 = C_1 \cdot \sum_{s \in S} \sum_{k \in K} M_{sdk} \qquad (6)$$

Soft restriction 1

- Optimal number of nurses
- RO: required optimum
- M: difference between optimal and actual

MATHEMATICAL MODEL

$$\sum_{d=d_0}^{d_f} \sum_{k \in K} x_{nsdk} + NMCAS_{nsd_0} \geq$$
$$MINCAS_s \cdot \sum_{k \in K} (x_{nsd_0k} - x_{n,s,d_0-1,k}), \tag{7}$$
$$\forall\ n \in N,\ s \in S,\ d_0 \in D \setminus \{1\},\ d_0 \leq |D| - MINCAS_s + 1,$$
$$d_f = d_0 + MINCAS_s - 1$$

$$\sum_{k \in K} BD_{nsk} + \sum_{k \in K} \sum_{d=1}^{d_f} x_{nsdk} + NMCAS_{n,s,1} \geq$$
$$MINCAS_s \cdot \sum_{k \in K} (x_{n,s,1,k} - IBD_{nsk}), \tag{8}$$
$$\forall\ n \in N,\ s \in S,\ d_f = MINCAS_s - BD_{nsk}$$

Soft restriction 2

- Consecutive assignments per shifts
- $NMCAS$: Number of Missing consecutive assignments
- $MINCAS$: Minimum Consecutive assignments
- BD: Border data

MATHEMATICAL MODEL

$$\sum_{d=d_0}^{d_f} \sum_{k \in K} x_{nsdk} - NECAS_{nsd_0} \leq MAXCAS_s, \tag{9}$$
$$\forall\ n \in N,\ \ s \in S,\ \ d_0 \in D \setminus \{1\},\ \ d_f = \min\{d_0 + MAXCAS_s, |D|\}$$

$$\sum_{k \in K} BD_{nsk} + \sum_{k \in K} \sum_{d=1}^{d_f} x_{nsdk} - NECAS_{n,s,1} \leq MAXCAS_s, \tag{10}$$
$$\forall\ n \in N,\ \ s \in S,\ \ d_f = MAXCAS_s - BD_{nsk} + 1$$

$$\Delta Z_2 = C_2 \cdot \sum_{n \in N} \sum_{s \in S} \sum_{d \in D} (NMCAS_{nsd} + NECAS_{nsd}) \tag{11}$$

Soft restriction 2

- $NECAS$: Number of Extra consecutive assignments
- $MAXCAS$: Maximum consecutive assignments

MATHEMATICAL MODEL

$$\sum_{d=d_0}^{d_f}\sum_{s=1}^{h}\sum_{k\in K} x_{nsdk} + NMCAG_{nd_0} \geq$$
$$MINCAG_n \cdot \sum_{s\in S}\sum_{k\in K}(x_{nsd_0k} - x_{n,s,d_0-1,k}), \quad (12)$$
$$\forall\ n \in N,\ d_0 \in D\setminus\{1\},\ d_0 \leq |D| - MINCAG_n + 1,$$
$$d_f = d_0 + MINCAG_n - 1$$

$$\sum_{s=1}^{h}\sum_{k\in K} BD_{nsk} + \sum_{s=1}^{h}\sum_{k\in K}\sum_{d=1}^{d_f} x_{nsdk} + NMCAG_{n,1} \geq$$
$$MINCAG_n \cdot \sum_{s=1}^{h}\sum_{k\in K} x_{n,s,1,k}, \quad (13)$$
$$\forall\ n \in N,\ d_f = MINCAG_n - BD_{nsk}$$

Soft restriction 3

- Overall working days
- $NMCAG$: number of Missing consecutive assignments globally
- $MINCAG$: Minimum consecutive assignments globally

MATHEMATICAL MODEL

$$\sum_{d=d_0}^{d_f} \sum_{s=1}^{h} \sum_{k \in K} x_{nsdk} - NECAG_{nd_0} \leq MAXCAG_n, \qquad (14)$$

$$\forall\ n \in N,\ d_0 \in D \setminus \{1\},\ d_f = \min\{d_0 + MAXCAG_n, |D|\}$$

$$\sum_{s=1}^{h} \sum_{k \in K} BD_{nsk} + \sum_{s=1}^{h} \sum_{k \in K} \sum_{d=1}^{d_f} x_{nsdk} - NECAG_{n,1} \leq MAXCAG_n, \qquad (15)$$

$$\forall\ n \in N,\ d_f = MAXCAG_n - BD_{nsk} + 1$$

$$\Delta Z_3 = C_3 \cdot \sum_{n \in N} \sum_{d \in D} (NMCAG_{nd} + NECAG_{nd}) \qquad (16)$$

- Soft restriction 3
- $NECAG$: number of Extra consecutive assignments globally
- $MAXCAG$: number of maximum Consecutive assignments globally

MATHEMATICAL MODEL

$$\Delta Z_4 = C_4 \cdot \sum_{n\in N}\sum_{s\in S}\sum_{d\in D}\sum_{k\in K} DS_{ns} \cdot x_{nsdk} \qquad (17)$$

$$\sum_{k\in K}\sum_{s=1}^{h} x_{nsdk} - MDW_{nd} \leq \sum_{k\in K}\sum_{s=1}^{h} x_{n,s,d-1,k} + (1 - W_n), \qquad (18)$$
$$\forall\ n \in N,\ d \in \{7, 14, 21, 28\}$$

$$MDW_{nd} + \sum_{k\in K}\sum_{s=1}^{h} x_{nsdk} \geq \sum_{k\in K}\sum_{s=1}^{h} x_{n,s,d-1,k} + (1 - W_n), \qquad (19)$$
$$\forall\ n \in N,\ d \in \{7, 14, 21, 28\}$$

$$\Delta Z_5 = C_5 \cdot \sum_{n\in N}\sum_{d\in D'} MDW_{nd}, \quad \text{where } D' = \{7, 14, 21, 28\} \qquad (20)$$

Soft restriction 4

- Preferences
- DS: Desire Satisfaction level

Soft restriction 5

- Working weekends
- MDW: missing days weekend
- W: work all weekend

MATHEMATICAL MODEL

$$\sum_{d\in D}\sum_{s=1}^{h}\sum_{k\in K} x_{nsdk} + NMWD_n \geq MINWD_n, \quad \forall\, n \in N \qquad (21)$$

$$\sum_{d\in D}\sum_{s=1}^{h}\sum_{k\in K} x_{nsdk} - NEWD_n \leq MAXWD_n, \quad \forall\, n \in N \qquad (22)$$

$$\Delta Z_6 = C_6 \cdot \sum_{n\in N} (NMWD_n + NEWD_n) \qquad (23)$$

Soft restriction 6

- Working days
- $NMWD$: number missing working days
- $MINWD$: minimum working days
- $NEWD$: number of extra working days
- $MAXWD$: maximum working days

MATHEMATICAL MODEL

$$\sum_{s=1}^{h}\sum_{k\in K}(x_{nsdk}+x_{n,s,d-1,k})\leq 2\cdot WW_{nd}$$
$$\forall\ n\in N,\ \ d\in\{7,14,21,28\} \tag{24}$$

$$\sum_{s=1}^{h}\sum_{k\in K}(x_{nsdk}+x_{n,s,d-1,k})\geq WW_{nd}$$
$$\forall\ n\in N,\ \ d\in\{7,14,21,28\} \tag{25}$$

$$\sum_{d\in D'}WW_{nd}-NEWW_{n}\leq MAXWW_{n},$$
$$\forall\ n\in N,\quad \text{where } D'=\{7,14,21,28\} \tag{26}$$

$$\Delta Z_7=C_7\cdot\sum_{n\in N}NEWW_{n} \tag{27}$$

Soft restriction 7

- Working weekends
- WW: Working weekend
- $NEWW$: Number of Extra working weekends
- $MAXWW$: Number of max working weekend

CONCLUSIONS AND FUTURE WORK

- Increasing difficulty
- Consideration of methods
- Implementation and experiments

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