A Multi-Start Iterative Local Search for the k-Traveling Repairman Problem

Research practice 2

Functional analysis and applications research group

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What is the problem?

- Given a set of activities and a set of resources, each activity must be made exactly once
- The resources cannot process activities simultaneously
- The weighted sum of start times is optimized
- The activities 0 and $n+1$ are dummy activities and they indicate the start and the end of the work

Why to do this?

- Industrial applications, such as machines scheduling and goods distribution.

- The complexity (np-hard) of the problems provides an appropriate field of research.

- The problems are part of the current state of the art in areas like the operations research, applied mathematics and optimization.

- The TRP approach is part of the models in humanitarian logistics, the humanitarian logistic is the organization of delivery and warehousing of supplies during complex emergencies .

Foto: UNICEF/HQ05-0041/Jeremy Horner

$$Min\ Z = \sum_{i=1}^{n} t_i \cdot w_i \qquad (1)$$

$$\sum_{i=0}^{n+1} x_{ij} = 1 \qquad \forall\ j \in J \backslash \{0, n+1\} \qquad (2)$$

$$\sum_{j=0}^{n+1} x_{ij} = 1 \qquad \forall\ i \in J \backslash \{0, n+1\} \qquad (3)$$

$$\sum_{j=1}^{n} x_{0j} = m \qquad (4)$$

$$t_j \geq t_i + s_{ij} + p_j - T \cdot (1 - x_{ij}) \qquad \forall\ i, j \in J \qquad (5)$$

$$x_{ij} \in \{0,1\} \qquad \forall\ i, j \in J \qquad (6)$$

$$t_j \geq 0 \qquad \forall\ j \in J \qquad (7)$$

- $\boldsymbol{x_{ij}}$: binary variable that indicates if the activity i is executed next the activity j
- $\boldsymbol{t_j}$**:** decision variable that represents the completion time of the activity j
- $\boldsymbol{s_{ij}}$**:** transition time between the activity i and activity j
- $\boldsymbol{p_j}$**:** time to process the activity j
- $\boldsymbol{w_i}$: represent the importance of the work I
- $\boldsymbol{T}$: Is the required time for plan, design and complete the activities.

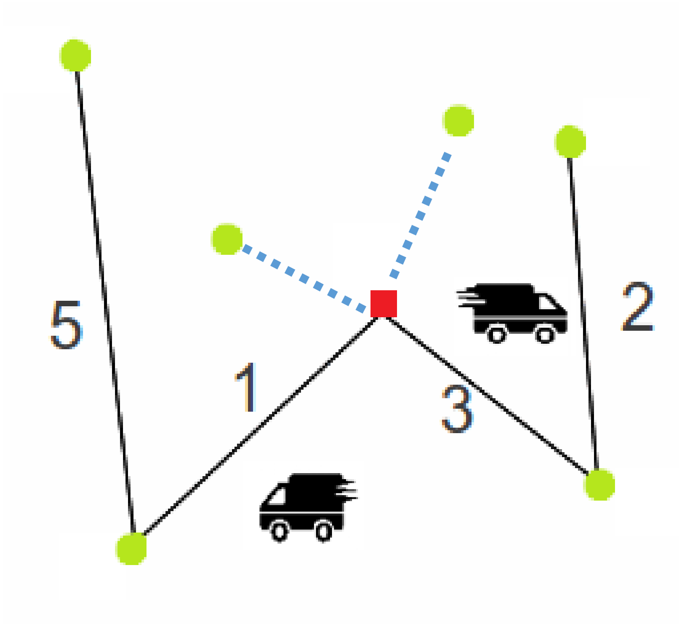
MS-ILS

- The Multi-Start Iterative Local Search (MS-ILS) is an iterative approach in which every iteration perform an ILS metaheuristic which starts with a random initial solution and improves that solution by a process which include perturbation and local search procedures.

Algorithm

- Random initial solution
- local search
- Perturbation
- Better solution

Constructive initial solution



- Is based on the nearest neighbor strategy.
- For each repairman, the algorithm chooses the two closest customers.
- the algorithm randomly assigns to each vehicle one of the two nearest

Precomputations

The procedure is performed to speed up the computation of the cost change by the subsequent local moves in the local search heuristic. This precomputations works as follows:

$$W_\sigma = \sum_{i=1}^{\sigma} w_{\sigma(i)} \qquad (7)$$

$$D_\sigma = \sum_{i=1}^{\sigma-1} d_{\sigma(i),\sigma(i+1)} \qquad (8)$$

$$C_\sigma = \sum_{i=1}^{\sigma-1} \left(d_{\sigma(i),\sigma(i+1)} \cdot \sum_{j=i+1}^{\sigma} w_{\sigma(j)} \right) \qquad (9)$$

The concatenation of sequence A and node B is defined as $A \oplus B$, and $W_{A \oplus B}$, $D_{A \oplus B}$, $C_{A \oplus B}$ can be computed by the following equations

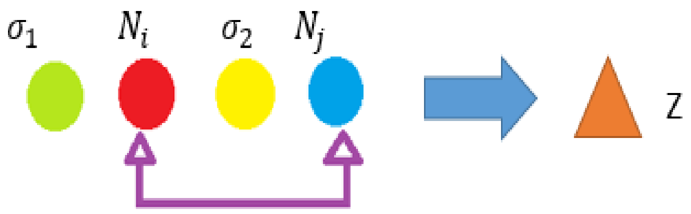
$$W_{A \oplus B} = W_A + w_B \quad (10)$$

$$D_{A \oplus B} = D_A + d_B \quad (11)$$

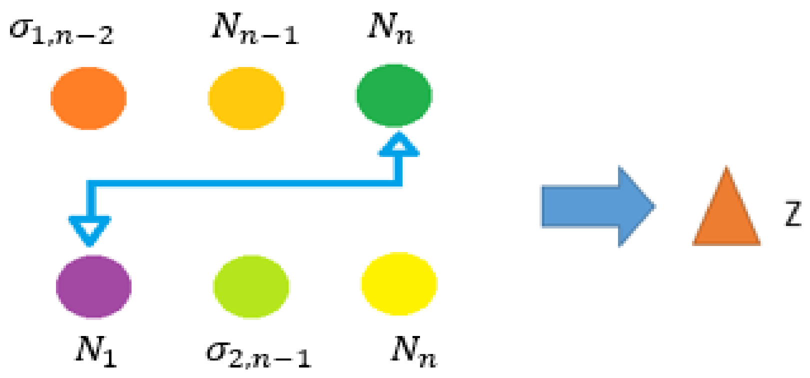
$$C_{A \oplus B} = C_A + w_B \, (D_A + d_B) + C_B \quad (12)$$

Local search

- the algorithm evaluate moves which exchange one customer from a route k_1 with another one of a route k_2. The best improvement strategy is applied



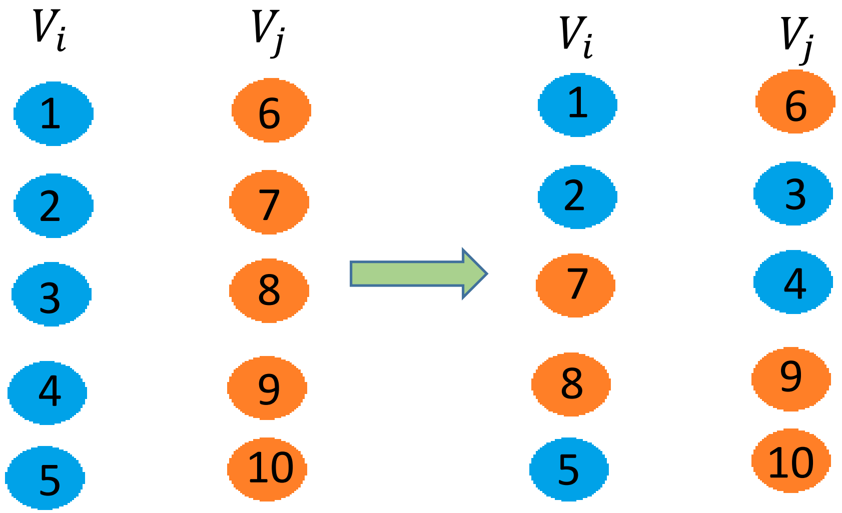
Same vehicle



Different vehicle

Perturbation

- The idea is to explore other solutions and diversify the search space.
- The algorithm choose a random number of customers between two random chosen vehicles.



Algorithm 1 Pseudo-code of the proposed MS-ILS

```
Read (data, starts, perturbations)
Z(bestSol) = ∞
for starts do
    initialSol = Constructive()
    Precomputations(initialSol)
    sol = LS(initialSol)
    for perturbations do
        pertSol = Pert(sol)
        newSol = LS(pertSol)
        if Z(newSol) < Z(sol) then
            sol = newSol
        end if
    end for
    if Z(sol) < Z(bestSol) then
        bestSol = sol
    end if
end for
return bestSol
```

Computational experiments

- For the performance assessment of the MS-LS, seven instances for the capacitated VRP are used, The instances vary the number of customers and the repairmen.

Instance	*1*	*2*	*3*	*4*	*5*	*11*	*12*
repairmen	5	10	8	12	17	7	10
customers	50	75	100	150	199	120	100

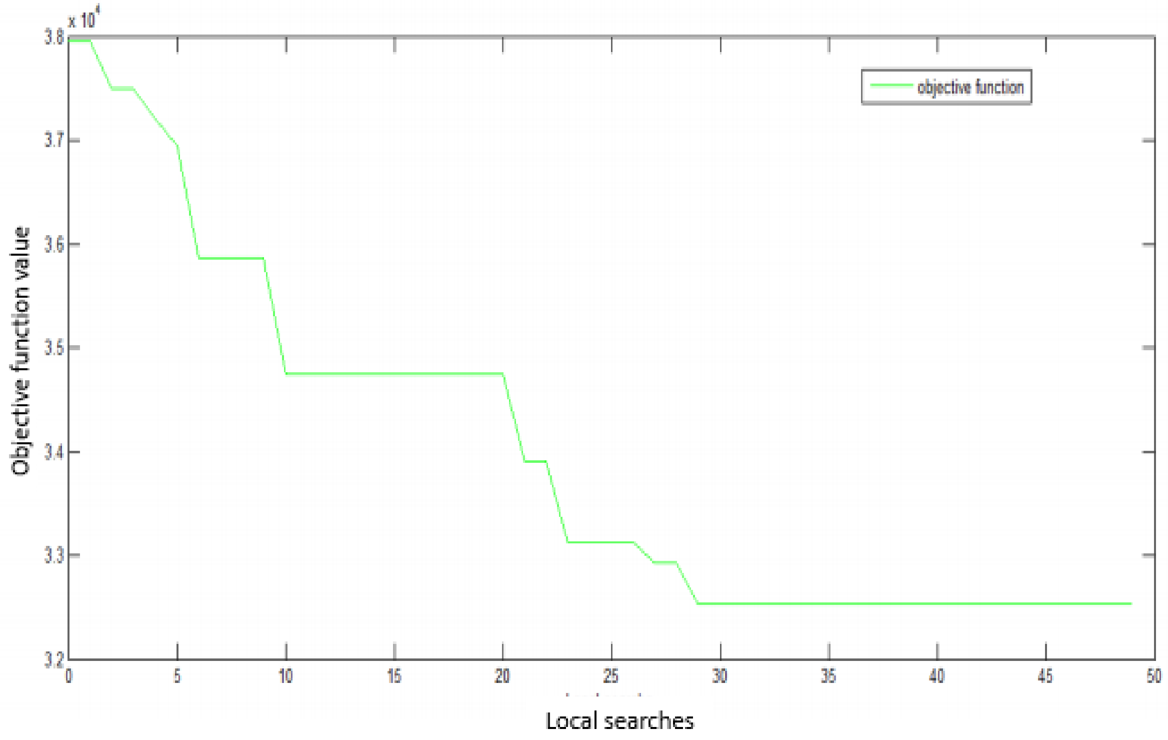
- An experimentation over the first instance with different values for both parameters was made to define the number of starts and the number of perturbations that the metaheuristic must execute

Type of instance	*Starts*	*Pert*	*Initial objective function value*	*Best objective function value*	*Improvement*	*Running time (sec)*
1	1	9999	3.82x10ˆ4	3,34x10ˆ4	4,81x10ˆ3	660
1	2	499	4,35x10ˆ4	3,35x10ˆ4	1x10ˆ4	593
1	5	199	4,08x10ˆ4	3,31x10ˆ4	7,64x10ˆ3	610
1	10	99	4,11x10ˆ4	3,39x10ˆ4	7,2x10ˆ3	684

Type of instance	*Starts*	*Pert*	*Initial objective function*	*Best objective function*	*Improvement*	*Running time (sec)*
1	5	199	4.1x10^4	3,35x10^4	7,52x10^3	590
2	5	199	5,16x10^4	4,3x10^4	8,57x10^3	1372
3	5	199	7,52x10^4	5,85x10^4	1,67x10^3	2952
4	5	199	1,03x10^5	8,09x10^3	● 2,22x10^4	6954
5	5	199	1,24x10^5	9,7x10^4	● 2,69x10^4	13268
6	5	199	1,57x10^5	6,04x10^4	● 9,69x10^4	9375
7	5	199	1,07x10^5	6,56x10^4	4,17x10^4	3174

Type of instance	Starts	*Pert*	*Initial objective function*	Best objective function	Improvement	Running time (sec)
1	2	499	4.33x10^4	3,23x10^4	● 1,1x10^4	581
2	2	499	5,46x10^4	4,21x10^4	● 1,25x10^4	1136
3	2	499	7,07x10^4	6,15x10^4	● 9,25x10^3	2594
4	2	499	9,34x10^4	7,69x10^4	1,65x10^4	6639
5	2	499	1,14x10^5	9,37x10^4	2,03x10^4	10.207
6	2	499	1,3x10^5	6,64x10^4	6,34x10^4	7172
7	2	499	1,27x10^5	6,48x10^4	● 6,18x10^4	2480

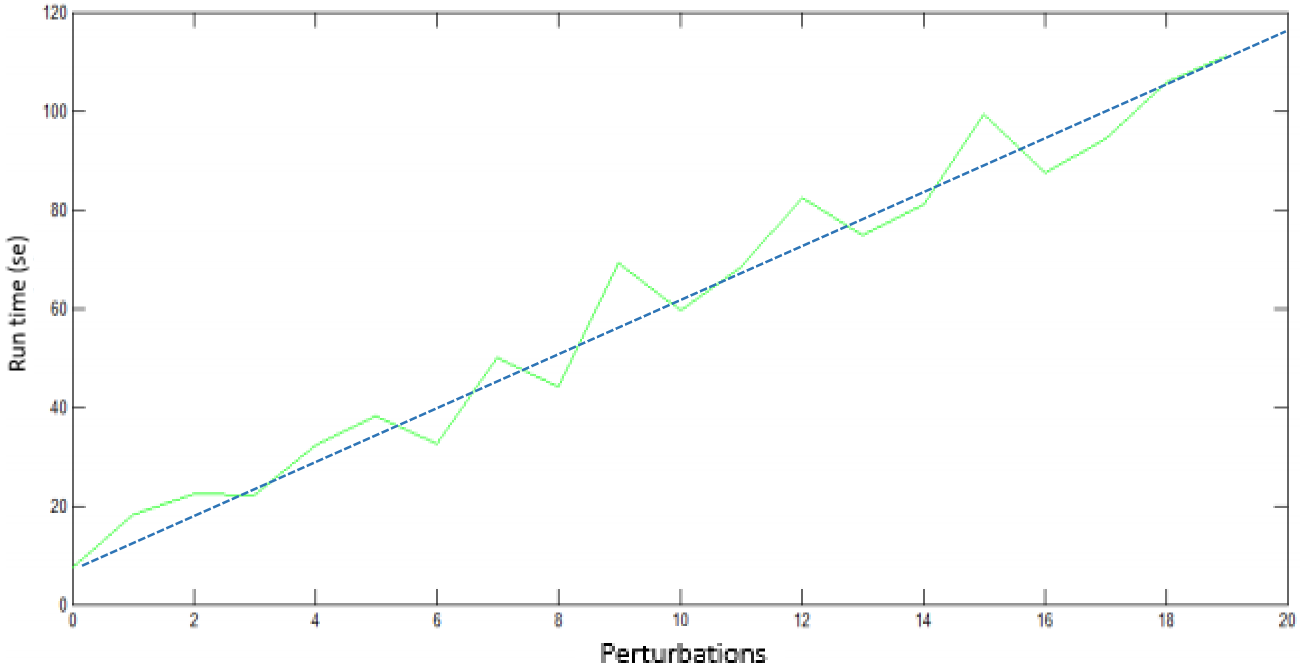
Improvements with LS



- the objective function value when different number of *local searches* are applied to the same problem.

- The objective function is less as more *local searches* are applied

Execution time



- the metaheuristic execution times when different number of perturbations are applied

- Taking into account that the times in this graphic are for different initial solutions, the behavior seems to be lineal.

Conclusions and further work

- The obtained results are not determinant about the parameters behavior, to conclude if some of the parameters are more important is necessary to test more instances.
- The improvements are relevant compared with the initial solution quality.
- As it was expected, the problems with more repairmen and nodes, require more time to get the solutions.
- Taking into account that parallel machine scheduling problems have the same mathematical formulation as further work, it is proposed to apply the MS-ILS to solve machines scheduling problems.

Thanks for your attention!