Ordinals and Typed Lambda Calculus

Course Introduction

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Administrative Information

Course web page

http://www1.eafit.edu.co/asr/courses/ordinals-and-typed-lambda-calculus/

Lectures dates, source code, etc.

See course web page.

Informally Building Sets

Definition

A set is **pure** iff its members are also sets.

Notation

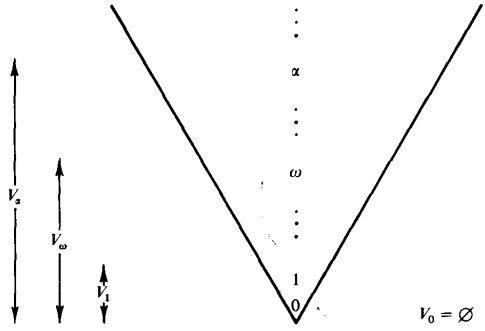
Let A be a set. The power set of A is denoted by $\mathrm{P}A$.

Convention

We shall use the terms ‘ordinal number’ and ‘ordinal’ interchangeably.

Informally Building Sets

The ordinal numbers are the backbone of the universe of (pure) sets*



$$V_0 \coloneqq \emptyset \qquad V_\omega \coloneqq V_0 \cup V_1 \cup \cdots \qquad V_{\alpha+1} \coloneqq \mathrm{P}V_\alpha$$
$$V_{n+1} \coloneqq \mathrm{P}V_n \qquad V_{\omega+1} \coloneqq \mathrm{P}V_\omega$$

*Figure source: [Enderton 1977, Fig. 3].

Classifying the Ordinals

$$\text{ordinals} \begin{cases} \text{zero ordinal} \\ \text{successor ordinal} \\ \text{limit ordinal} \end{cases}$$

$$\text{ordinals} \begin{cases} \text{countable} \begin{cases} \text{computable } (\lambda\text{-definable or constructive}) \\ \text{incomputable} \end{cases} \\ \text{uncountable} \end{cases}$$

References

 Enderton, Herbert B. (1977). Elements of Set Theory. Academic Press (cit. on p. 4).