# Ordinals and Typed Lambda Calculus 

Course Introduction

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## Administrative Information

Course web page
http://www1.eafit.edu.co/asr/courses/ordinals-and-typed-lambda-calculus/
Lectures dates, source code, etc.
See course web page.

## Informally Building Sets

## Definition

A set is pure iff its members are also sets.

## Notation

Let $A$ be a set. The power set of $A$ is denoted by $\mathrm{P} A$.

Convention
We shall use the terms 'ordinal number' and 'ordinal' interchangeably.

## Informally Building Sets

The ordinal numbers are the backbone of the universe of (pure) sets*


$$
\begin{aligned}
V_{0} & :=\emptyset & V_{\omega} & :=V_{0} \cup V_{1} \cup \cdots \\
V_{n+1} & :=\mathrm{P} V_{n} & V_{\omega+1} & :=\mathrm{P} V_{\omega}
\end{aligned}
$$

*Figure source: [Enderton 1977, Fig. 3].

## Classifying the Ordinals

ordinals $\left\{\begin{array}{l}\text { zero ordinal } \\ \text { successor ordinal } \\ \text { limit ordinal }\end{array}\right.$
ordinals $\left\{\begin{array}{l}\text { countable }\left\{\begin{array}{l}\text { computable }(\lambda \text {-definable or constructive }) \\ \text { incomputable }\end{array}\right. \\ \text { uncountable }\end{array}\right.$

## References

Enderton, Herbert B. (1977). Elements of Set Theory. Academic Press (cit. on p. 4).

