CM0845 Logic
Propositional Logic: Satisfiability

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Remark

The reference for this section is Ben-Ari [2012, § 2.5].

Satisfiability, Validity, Unsatisfiability and Falsifiability

Let $\varphi \in \mathrm{PROP}$.

Definitions

(i) φ is **satisfiable** iff $[\![\varphi]\!]_v = 1$ for some interpretation v.
In this case, v is called a model for φ.

(ii) φ is **valid** (a tautology), denoted $\models \varphi$, iff $[\![\varphi]\!]_v = 1$ for all interpretations v.

(iii) φ is **unsatisfiable** iff it is not satisfiable, that is, if $[\![\varphi]\!]_v = 0$ for all interpretations v.

(iv) φ is **falsifiable**, denoted $\not\models$, iff it is not valid, that is, if $[\![\varphi]\!]_v = 0$ for some interpretation v.

Satisfiability, Validity, Unsatisfiability and Falsifiability

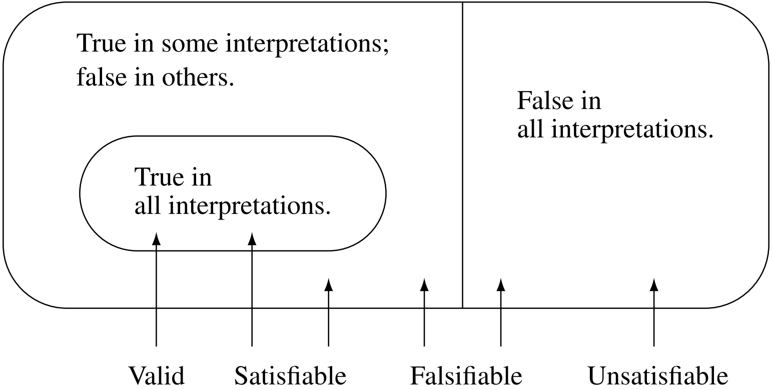

Figure 2.6 of [Ben-Ari 2012].

Satisfiability, Validity, Unsatisfiability and Falsifiability

Theorem (Ben-Ari [2012], Theorem 2.39)

Let $\varphi \in \mathrm{PROP}$.

(i) The proposition φ is valid if and only if $\neg\varphi$ is unsatisfiable.

(ii) The proposition φ is satisfiable if and only if $\neg\varphi$ is falsifiable.

Satisfiability of a Set of Propositions

Let $\Gamma = \{\varphi_1, \dots\}$ be a set of propositions.

Definitions

(i) Γ is **satisfiable** iff there exists an interpretation v such that $[\![\varphi]\!]_v = 1$ for all $\varphi_i \in \Gamma$. In this case, v is a model of Γ.

(ii) Γ is **unsatisfiable** iff for every interpretation v, there exists an $\varphi_i \in \Gamma$ such that $[\![\varphi]\!]_v = 0$.

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Example

Prove that if Γ is unsatisfiable and for some i, the proposition φ_i is valid, then $\Gamma - \{\varphi_i\}$ is unsatisfiable [Ben-Ari 2012, Exercise 2.15, p. 46].

References

Ben-Ari, Mordechai [1993] (2012). Mathematical Logic for Computer Science. 3rd ed. Springer (cit. on pp. 2, 4–7).