# Logic - CM0845 Introduction to Haskell 

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## What is Haskell?

Haskell is a purely functional programming language. That means that every function in Haskell is also a function in the mathematical sense.

## Example

```
factorial 0 = 1
factorial n = n * factorial (n - 1)
```


## Functions

## Example

```
factorial \(0=1\)
factorial \(n=n\) * factorial (n - 1)
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factorial :: Int -> Int
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## Functions

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factorial :: Int -> Int
factorial 0 = 1
factorial \(n=n\) * factorial (n - 1)
```

But -1 is an Integer, so...

## Functions

## A solution for this bug:

```
factorial :: Int -> Int
factorial n
    \(\mid \mathrm{n}=0=1\)
    | \(\mathrm{n}>0=\mathrm{n}\) * factorial (n - 1)
    | otherwise = error "factorial: n < 0"
```


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```

There are more than you believe!
Google for "The evolution of a Haskell programmer".

## Lists

Inductive definition Haskell has a built-in syntax for lists, where a list is either:

- the empty list, written [ ], or
- an element $\mathbf{x}$ and a list $\mathbf{x s}$, written ( $\mathbf{x}: \mathbf{x s}$ ).


## Lists

## Example - Pattern matching on lists

```
length :: [Int] -> Int
length [] = 0
length (x : xs) = 1 + length xs
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What if one wanted to get the length of a list of Booleans?

```
length :: [Bool] -> Int
length [] = 0
length (x : xs) = 1 + length \(x s\)
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## Lists

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What if one wanted to get the length of a list of Booleans?

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length :: [Bool] -> Int
length [] = 0
length (x : xs) = 1 + length xs
```

Take it easy, there's another solution!

## Parametric Polymorphism

## Example - Basic functions

-- Returns the length of a finite list as an Int. length :: [a] -> Int
-- Appends two lists.
(++) :: [a] -> [a] -> [a]
-- Extracts the first element of a list.
head :: [a] -> a
-- Extracts the last element of a list.
last :: [a] -> a

## Example - Basic functions

-- Extracts the elements after the head of a list. tail :: [a] -> [a]
-- Returns all the elements of a list except -- the last one.
init :: [a] -> [a]
-- Testes if a list is empty.
null :: [a] -> Bool

## Lazy

## Haskell wont't execute functions or calculate things until necessary.

## Example

```
foo :: Int -> Bool -- Non-terminating function.
foo n = foo (n + 1)
bar :: Int -> Bool
bar n = True || foo n
bar' :: Int -> Bool
bar' n = foo n || True
```


## Lazy

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Try to calculate bar 3.

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Try to calculate bar 3.
Try to calculate bar' 3.

## High-Order Functions and Currying

Every function in Haskell officially only takes one parameter. So how can we define a function that takes more than a parameter?

```
-- Takes two things that can be ordered and
returns the greater one.
\(\max ::(\) Ord \(a)=>a \quad->a \quad->a\)
```


## Example

- max 23


## High-Order Functions and Currying

Every function in Haskell officially only takes one parameter. So how can we define a function that takes more than a parameter?

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max :: (Ord a) => a -> a -> a
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## Example

- max 23
- (max 2) 3


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## Example

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- (max 2) 3

Haskell functions can take functions as parameters and return functions as return values!

## High-Order Functions

## Example

```
-- map f xs is the list obtained by applying f
-- to each element of xs.
map :: (a -> b) -> [a] -> [b]
map f [] = []
map f (x : xs) = f x : map f xs
```

Which is the value of map $(* 2) \quad[1,2,4] ?$

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## Example

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-- map f xs is the list obtained by applying f
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Which is the value of map $(* 2) \quad[1,2,4] ?$
GHCi> map (*2) [1, 2, 4]
$[2,4,8]$

## Example

-- foldr, applied to a binary operator, a starting
-- value and a list, reduces the list using th
-- binary operator, from right to left (see also
-- foldl):
-- foldr f $z$ [x1, $x 2, \ldots, x n]==$
-- x1 `f` (x2 `f'... (xn `f` z)...)
foldr :: (a -> b -> b) -> b -> [a] -> b
foldr f z [] = z
foldr $f$ z (x : xS) $=f$ x (foldr f $z$ xs)

## Example

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foldr :: (a -> b -> b) -> b -> [a] -> b
foldr f z [] = z
foldr f $z(x: x s)=f x$ (foldr $f \quad z \quad x S)$

GHCi> foldr (*) 1 [1..5]
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## Creating Types - Algebraic Data Types

## Example

```
data Bool = True | False
```

Functions by pattern-matching

## Creating Types - Algebraic Data Types

## Example

```
-- Recursive data type.
data Nat = Zero | Succ Nat
```

Functions by pattern-matching

```
(+) :: Nat -> Nat -> Nat
Zero + n = n
(Succ m) + n = Succ (m + n)
```


## Example

```
-- Polymorphic data type.
data List a = Nil | Cons a (List a)
```


## Some Links

- Real-World Applications

See http://www.haskell.org/haskellwiki/Haskell_ in_industry.

- Nice Tutorial

See http://learnyouahaskell.com.

- Downloading

See https://www.haskell.org/downloads.

