CM0832 Elements of Set Theory
8. Ordinals and Order Types

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Preliminaries

Convention

The number assigned to chapters, examples, exercises, figures, sections, and theorems on these slides correspond to the numbers assigned in the textbook [Enderton 1977].

Ordinal Addition

Definition

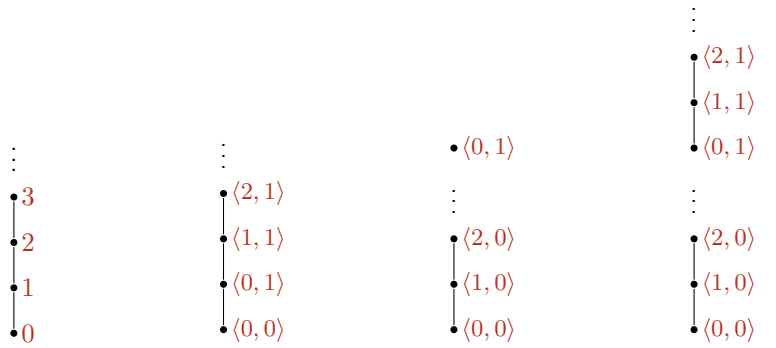
Let α and β be ordinals. We define their **addition** by transfinite recursion on β:

$$\begin{aligned}
\alpha + 0 &= \alpha, \\
\alpha + \beta^+ &= (\alpha + \beta)^+, \\
\alpha + \beta &= \sup \{ \alpha + \lambda \mid \lambda < \beta \} \\
&= \bigcup_{\lambda < \beta} (\alpha + \lambda), \text{ for all limit ordinal } \beta.
\end{aligned}$$

Ordinal Addition

Example

$\omega = 1 + \omega$, $1 + \omega \neq \omega + 1$ and $\omega + 1 \neq \omega + \omega$.



(a) ω (b) $1 + \omega$ (c) $\omega + 1$ (d) $\omega + \omega$

References

 Enderton, Herbert B. (1977). Elements of Set Theory. Academic Press (cit. on p. 2).