CM0832 Elements of Set Theory
Cardinal Numbers

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Preliminaries

Convention

The number assigned to chapters, examples, exercises, figures, sections, and theorems on these slides correspond to the numbers assigned in the textbook [Hrbacek and Jech (1978) 1999].

Cardinal Arithmetic

Definition

The **sum** of cardinal numbers is defined by

$$\kappa + \lambda := |A \cup B|,$$

where A and B are two arbitrary sets such that $|A| = \kappa$, $|B| = \lambda$, and $A \cap B = \emptyset$.

Cardinal Arithmetic

Definition

The **multiplication** of cardinal numbers is defined by

$$\kappa \cdot \lambda := |A \times B|,$$

where A and B are two arbitrary sets such $|A| = \kappa$ and $|B| = \lambda$.

Cardinal Arithmetic

Definition

The **exponentiation** of cardinal numbers is defined by

$$\kappa^\lambda := |A^B|,$$

where A and B are two arbitrary sets such $|A| = \kappa$ and $|B| = \lambda$.

The Cardinality of the Continuum

Theorem (Ch. 5, Theorem 2.1)

$|\mathbb{R}| = 2^{\aleph_0}$.

The Cardinality of the Continuum

Theorem (Ch. 5, Theorem 2.2.a)

Let $n \in \mathbb{N}$, then

$$\begin{aligned} n + 2^{\aleph_0} &= \aleph_0 + 2^{\aleph_0} \\ &= 2^{\aleph_0} + 2^{\aleph_0} \\ &= 2^{\aleph_0}. \end{aligned}$$

The Cardinality of the Continuum

Theorem (Ch. 5, Theorem 2.2.b)

Let $n \in \mathbb{N}$ and $n > 0$, then

$$\begin{aligned} n \cdot 2^{\aleph_0} &= \aleph_0 \cdot 2^{\aleph_0} \\ &= 2^{\aleph_0} \cdot 2^{\aleph_0} \\ &= 2^{\aleph_0}. \end{aligned}$$

The Cardinality of the Continuum

Theorem (Ch. 5, Theorem 2.2.c)

Let $n \in \mathbb{N}$ and $n > 0$, then

$$\begin{aligned}(2^{\aleph_0})^n &= (2^{\aleph_0})^{\aleph_0} \\ &= n^{\aleph_0} \\ &= \aleph_0^{\aleph_0} \\ &= 2^{\aleph_0}.\end{aligned}$$

References

Hrbacek, Karel and Jech, Thomas [1978] (1999). Introduction to Set Theory. Third Edition, Revised and Expanded. Marcel Dekker (cit. on p. 2).