CM0081 Automata and Formal Languages
Hypercomputation

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Motivation

Absolute computability

> *'For how can we ever exclude the possibility of our being presented, some day (perhaps by some extraterrestrial visitor), with a (perhaps extremely complex) device or "oracle" that "computes" a noncomputable function? However, there are fairly convincing reasons for believing that this will never happen.' [Davis 1958, p. 11]*

Motivation

Relative computability

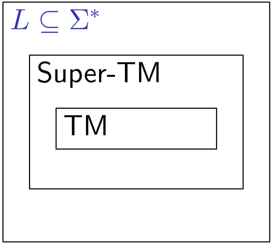
> *'Troubles with absolutism are deeper and more extensive than these cracks (analogue procedures and newer physics) reveal. For one thing, computability is relative not simply to physics, but more generally to systems of frameworks, which include or contain underlying logics.' [Sylvan and Copeland 2000, p. 190]*

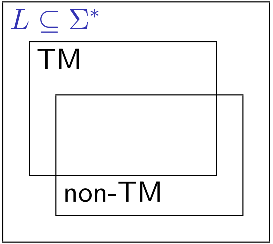
Hypercomputers

Definition

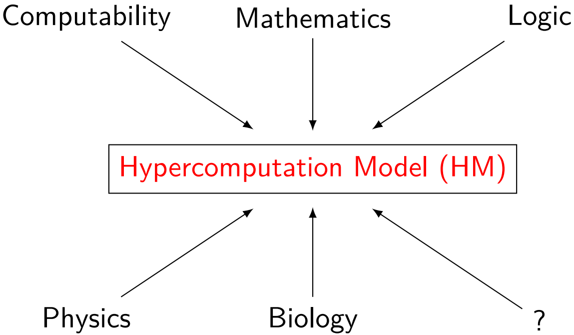
> ‘***Hypercomputation*** *is the computation of functions or numbers that cannot be computed in the sense of Turing [1936–1937], i.e., cannot be computed with paper and pencil in a finite number of steps by a human clerk working effectively.’ [Copeland 2002b, p. 461]*

Super Turing Machines and Non Turing Machines





Possible Sources of Hypercomputation



First Hypercomputation Model: Oracle Turing Machines

Definition

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Hypercomputability features

- If the oracle is a recursive set then OTM $\equiv$ TM.
- If the oracle is a non-recursive set then OTM $\equiv$ HM.

On the 'Hypercomputation' Term

Copeland and Proudfoot [1999]:

Right	Wrong
Hypercomputation	Super-Turing computation
	Computing beyond Turing's limit
	Breaking the Turing barrier
	Etc.

Hypercomputation Model: Accelerated Turing Machines

Definition

An **accelerated Turing machine** (ATM) is a Turing machine that performs its first step in one unit of time and each subsequent step in half the time of the step before [Copeland 1998, 2002a].

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Hypercomputability features

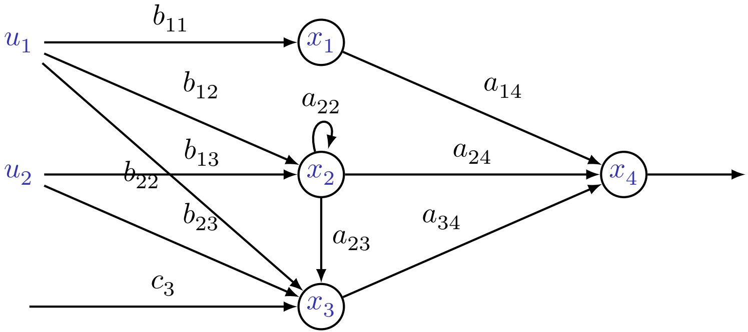
Since

$$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots = \sum_{i=0}^{\infty} \frac{1}{2^i} = 2,$$

an ATM could complete an infinity of steps in two time units.

Hypercomputation Model: Analog Recurrent Neural Network (ARNN)

Description [Siegelmann 1999]



$$\overrightarrow{X}(t+1) = \sigma\left(\overrightarrow{A}\cdot\overrightarrow{X}(t) + \overrightarrow{B}\cdot\overrightarrow{U}(t) + \overrightarrow{C}\right)$$

Hypercomputation Model: Analog Recurrent Neural Network (ARNN)

Hypercomputability features

$$a_{ij} \in \{\mathbb{N}, \mathbb{Q}, \mathbb{R}\} \Rightarrow \text{ARNN} \equiv \{\text{DFA}, \text{TM}, \text{HM}\}.$$

Standard Quantum Computation (SQC)

Models

Quantum Turing machines (QTM) [Deutsch 1985] and quantum circuits [Deutsch 1989].

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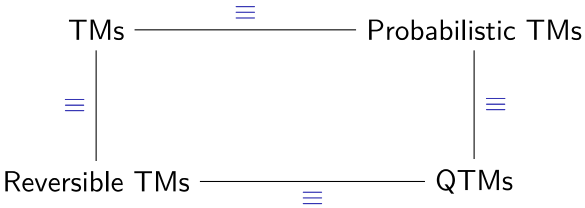
Relation between the models



'Weak' Hypercomputation Based on SQC

Generation of truly random numbers

$$U_H \,|\,0\rangle = \frac{1}{\sqrt{2}}(|\,0\rangle + |\,1\rangle) \rightarrow \text{measure}$$

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1. We observe the superposition state: *'The act of observation causes the superposition to collapse into either* $|\,0\rangle$ *or the* $|\,1\rangle$ *state with equal probability. Hence you can exploit quantum mechanical superposition and indeterminism to simulate a perfectly fair coin toss.' [Williams and Clearwater 1997, p. 160]*

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1. We observe the superposition state: *‘The act of observation causes the superposition to collapse into either $|\,0\rangle$ or the $|\,1\rangle$ state with equal probability. Hence you can exploit quantum mechanical superposition and indeterminism to simulate a perfectly fair coin toss.’ [Williams and Clearwater 1997, p. 160]*
2. The problem: It is not clear how to use this property to solve a non-computable Turing machine problem [Ord and Kieu 2009].

Others Quantum Computation Models

Common misunderstanding

$$\begin{aligned}\text{quantum computation} &\equiv \text{SQC}\\ &\equiv \text{adiabatic quantum computation (AQC)}\end{aligned}$$

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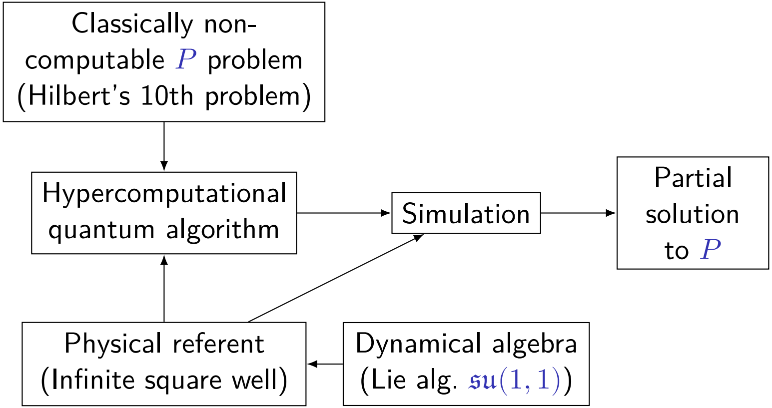
The real situation

Kieu's hypercomputational quantum algorithm (KHQA) [Kieu 2003a]:

$$\begin{aligned}\text{finite AQC} &\equiv \text{SQC}\\ \text{infinite AQC} &\equiv \text{KHQA}\end{aligned}$$

Hypercomputational Quantum Algorithm à la Kieu

Sicard, Ospina and Vélez [2006]:



Hypercomputation Model: Infinite Time Turing Machines

Definition

An **infinite time Turing machine** is a Turing machine working on a time clocked by transfinite ordinals [Hamkins and Lewis 2000; Hamkins 2002, 2007].

†Figure from Hamkins [2002, Fig. 1].

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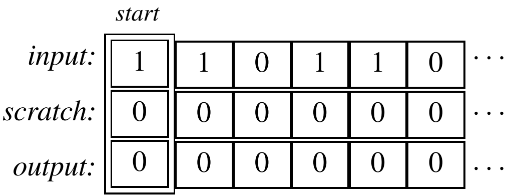
Description

For convenience, the machines have three tapes:†



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Hypercomputation Model: Infinite Time Turing Machines

Description (continuation)

In stage $\alpha + 1$ the machine works as usual.

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In limit ordinal stages the machine works as follows [Hamkins and Lewis 2000, p. 569–570]:

> *'To set up such a limit ordinal configuration, the head is plucked from wherever it might have been racing towards, and placed on top of the first cell. And it is placed in a special distinguished limit state.'*

> *'Now we need to take a limit of the cell values on the tape. And we will do this cell by cell according to the following rule: if the values appearing in a cell have converged, that is, if they are either eventually 0 or eventually 1 before the limit stage, then the cell retains the limiting value at the limit stage. Otherwise, in the case that the cell values have alternated from 0 to 1 and back again unboundedly often, we make the limit cell value 1.'*

Description (continuation)

> *'This completely describes the configuration of the machine at any limit ordinal stage* β*, and the machine can go on computing,* $\beta + 1$*,* $\beta + 2$*, and so on, eventually taking another limit at* $\beta + \omega$ *co and so on through the ordinals.'*

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Hypercomputability features

The halting problem is decidable in ω many steps by infinite time Turing machines [Hamkins and Lewis 2000].

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Observation

Although not related with computability but algorithmic complexity, $\mathrm{P} \neq \mathrm{NP}$ for infinite time Turing machines [Schindler 2003].

Hypercomputation Model: Infinite Time Turing Machines

Theorem (Hamkins and Lewis [2000, Theorem 1.1])

Every halting infinite time computation is countable.

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Observation

The infinite time Turing machines have been generalised by the **ordinal computability models**, which are models based on ordinal numbers. These models include infinite time Turing machines or Turing machines working on tapes of transfinite 'length'. Seyfferth [2013] shows a brief overview of these models.

Church-Turing Thesis and Thesis M

The Church-Turing thesis

> *'Any procedure than can be carried out by an idealised human clerk working mechanically with paper and pencil can also be carried out by a Turing machine.' [Copeland and Sylvan 1999]*

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Thesis M

'What can be calculated by a machine is Turing machine computable.' [Gandy 1980]

Physical Hypercomputation?

Open problem

The refutation of a general/physical version of Gandy's Thesis M.

Physical Hypercomputation?

Proposals?

[†] See [Ord and Kieu 2009; Copeland 2002b].

[‡] See [Sicard, Ospina and Vélez 2006; Kieu 2005, 2004b,a, 2003b,a, 2002].

[§] See [Copeland and Shagrir 2020; Andréka, Madarász, I. Németi, P. Németi and Székely 2018; Welch 2008; I. Németi and Dávid 2006; Hogarth 2004; Etesi and I. Németi 2002; Hogarth 1994, 1992; Pitowsky 1990].

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Physical Hypercomputation?

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- Hypercomputation based on oracles.[†]

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- Hypercomputation based on coherent domains.[‖]

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An Interesting Project: Formal Verification of Hypercomputation in Relativistic Physics

Stannett and I. Németi [2014] and Stannett [2015]:

Goals

- Implement first-order axiomatisations of theories of the relativity using the proof assistant ISABELLE;

An Interesting Project: Formal Verification of Hypercomputation in Relativistic Physics

Stannett and I. Németi [2014] and Stannett [2015]:

Goals

- Implement first-order axiomatisations of theories of the relativity using the proof assistant Isabelle;
- Add a model of computation carried out by machines travelling along specific spacetime trajectories;

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An Interesting Project: Formal Verification of Hypercomputation in Relativistic Physics

Goals (continuation)

- Consider how the power of these computational systems changes according to the underlying topology of spacetime;

An Interesting Project: Formal Verification of Hypercomputation in Relativistic Physics

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- Consider how the power of these computational systems changes according to the underlying topology of spacetime;
- Select a recursively uncomputable problem P (e.g. the Halting Problem) and machine-verify the following claims:
  - in simpler relativistic settings, P remains uncomputable;
  - in some spacetimes, P can be solved.

An Interesting Project: Formal Verification of Hypercomputation in Relativistic Physics

Some formalisations on Isabelle

(i) The Halting Problem is Soluble in Malament-Hogarth Spacetimes [Stannett 2023].

(ii) No Faster-Than-Light Observers (GenRel) [Stannett, Higgins, Andreka, Madarász, I. Németi and Székely 2023].

(iii) No Faster-Than-Light Observers [Stannett and I. Németi 2016].

Is Hypercomputation a Myth?

Davis' refutations

- Davis, M. [2006]. Why There is no Such Discipline as Hypercomputation. Applied Mathematics and Computation 178.1, pp. 4–7. DOI: 10.1016/j.amc.2005.09.066.

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Refutation to Davis

Sundar, N. and Bringsjord, S. [2011]. The Myth of 'The Myth of Hypercomputation'. In: Combined Pre-Proceedings of P&C 2011 and HyperNet 2011. Ed. by Stannett, M., pp. 185–196.

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Final Remarks

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'We live in a quantum-mechanical, relativistic physical universe, with bizarre physical phenomena that we are only beginning to understand. Perhaps we might hope to take computational advantage of some strange physical effect. Perhaps the physical world is arranged in such a way that allows for the computation of a non-Turing-computable function by means of some physical procedure.' [Hamkins 2021, § 6.4]

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Final Remarks

'Is there any limit to discrete computation, and more generally, to scientific knowledge?' [Calude and Dinneen 1998, p. 1]

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'"In breaking the Turing barrier, our knowledge of the world, and therefore our control of it, would be altered forever," Professor Cooper added.' [Computing a way through the Turing barrier. The Reporter. The University of Leeds Newsletter. No. 505. 21 February 2005.]

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