Section 5.4
Binomial Coefficients

Pascal's Identity:

$$C(n+1,k) = C(n,k-1) + C(n,k)$$

Proof:

We construct subsets of size k from a set with n + 1 elements given the subsets of size k and k-1 from a set with n elements.

The total will include

• all of the subsets from the set of size n which do not contain the new element

C(n, k),

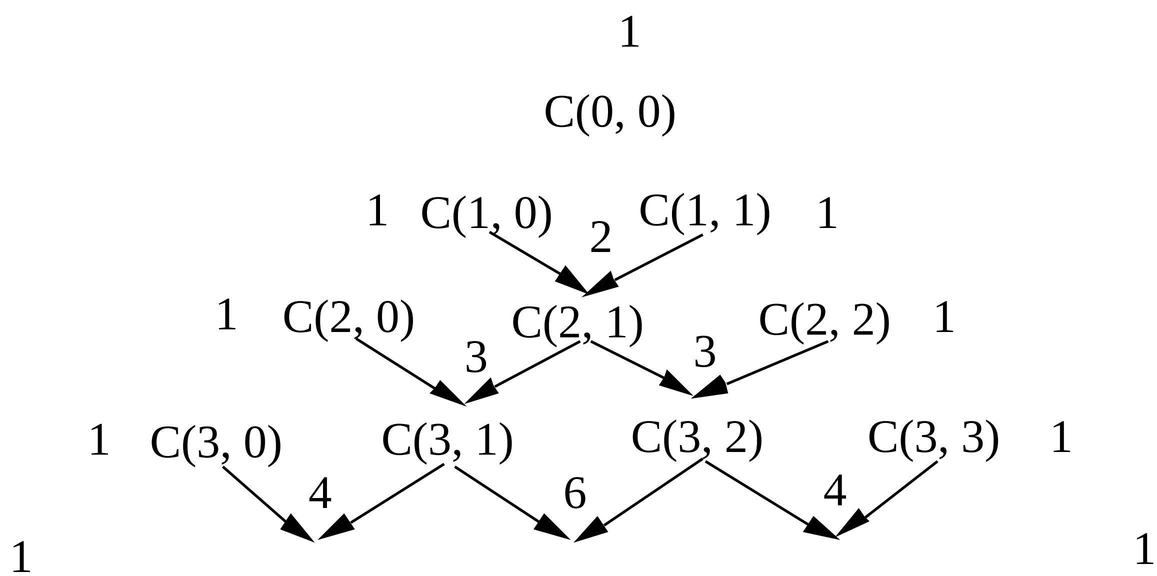
plus

• the subsets of size k - 1 with the new element added

C(n, k-1).

It produces

Pascal's triangle



A good way to evaluate C(n, r) for large n and r (to avoid overflow).

Example:

How many bit strings of length 4 have exactly 2 ones (or exactly 2 zeros)?

Analysis:

We solve the problem by determining the positions of the two ones in the bit string.

- place the first one - 4 possibilities
- place the second one - 3 possibilities

Hence it appears that we have (4)(3)= 12 possibilities.

We enumerate them to make sure:

0011, 0101, 1001, 0110, 1010, 1100.

There are actually only 6 possibilities. What is wrong?

The answer would be correct if we had two <u>different</u> objects to place in the string.

For example, if we were going to place an ‘a’ and a ‘b’ in the string we would have

00ab, 00ba, 0a0b, 0b0a, a00b, b00a,

and so forth for a total of 12.

But......the objects (1 and 1) are the same <u>so the order is not important</u>!

Divide through by the number of orderings = 2! = 2.

Therefore the answer is 12/2 = 6.

Example:

How many bit strings of length 4 have at least 2 ones?

Analysis:

Total the number of strings that have

- zero 1's = 1
- one 1 = 4

Total = $2^4 - 5 = 11$.

If the universe is the bit strings of length 4, what is the complement of the above set?

What is its cardinality?
